4) *If I is a 1-solvable ideal of V with $I^{(s,1)} = \{0\}$ with s minimal, then $I^{(i,1)} \subseteq C_{s-i}(V)$ for all $0 \leq i \leq s$.*

To answer the question which connections there exist among the k-solvabilities of ideals we prove the inclusion $I^{(s,k')} \subseteq I^{(s,k)}$ for $k \leq k'$. The case $s = 0$ is clear, since $I^{(0,k)} = I^{(0,k')} = I$. Suppose now that the inclusion is true for s, we proceed to $s+1$. By definition,

$$\begin{aligned} I^{(s+1,k')} &= [\underbrace{I^{(s,k')}, \cdots, I^{(s,k')}}_{k'}, V, \cdots, V] \\ &\subseteq [\underbrace{I^{(s,k)}, \cdots, I^{(s,k)}}_{k'}, V, \cdots, V] \\ &\subseteq [\underbrace{I^{(s,k)}, \cdots, I^{(s,k)}}_{k}, V, \cdots, V] \\ &= I^{(s+1,k)}. \end{aligned}$$

If I is a k-solvable ideal, then $I^{(s,k)} = \{0\}$ for some $s \in \mathbb{N}$, therefore $I^{(s,k')} = \{0\}$, that is, I is a k'-solvable ideal by definition. Analogously we can show that a k-solvable subalgebra of V is also a k'-solvable subalgebra of V.

Proposition 2.1: *Let V be an n-Lie algebra and $1 \leq k \leq k' \leq n$. A k-solvable ideal (subalgebra) of V is also a k'-solvable ideal (subalgebra) of V. In particular, the k-solvability implies n-solvability for all k.*

Example 2.1: One can show that an $(n+1)$-dimensional space V with base $\{e_1, \cdots, e_{n+1}\}$ and multiplication defined by the formulas $[e_1, \cdots, \widehat{e_i}, \cdots, e_{n+1}] = \alpha_i e_i$, $\alpha_i \in K$, $i \in \underline{n+1}$, is an n-Lie algebra for any choice of the constants α_i (cf. Filippov [5]). Now suppose $\alpha_1 \cdots \alpha_k \neq 0$, $\alpha_{k+1} = \cdots = \alpha_{n+1} = 0$, $2 \leq k \leq n$. Then the ideal $I := V^{(1,k)} = [V, \cdots, V]$ has base $\{e_1, \cdots, e_k\}$ and $V^{(2,k)} = [\underbrace{I, \cdots, I}_{k}, V, \cdots, V] = \{0\}$, hence V is a k-solvable n-Lie algebra and I a k-solvable ideal of V. Since $[I, \cdots, I] = \{0\}$, I is l-solvable subalgebra of V for all $1 \leq l \leq n$. Further we can check that $V^{(2,k-1)} = [\underbrace{I, \cdots, I}_{k-1}, V, \cdots, V] = I$, hence V is not $(k-1)$-solvable and I is not a $(k-1)$-solvable ideal of V (I is also not l-solvable ideal of V for all $l \leq k-1$ by Proposition 2.1).

Indeed, V has no nonzero $(k-1)$-solvable ideal if $k \geq 3$. To show this we prove that each nonzero ideal of V contains I. Let J be an arbitrary nonzero ideal of V and $0 \neq v \in J$, $v = \sum_{j=1}^{n+1} \beta_j e_j$, $\beta_j \in K$. Then there is a $\beta_{j_0} \neq 0$. Let $i \leq k$, $i \neq j_0$ (such an i exists because $k \geq 3$), then

$$J \ni [e_1, \cdots, \widehat{e_i}, \cdots, e_{j_0-1}, v, e_{j_0+1}, \cdots, e_{n+1}]$$