isomorphism:

$$\wedge^n \overline{V} \cong \sum_{\substack{0 \le n_1, \cdots, n_m \le n \\ n_1 + \cdots + n_m = n}} \wedge^{n_1} \overline{V}_1 \otimes \cdots \otimes \wedge^{n_m} \overline{V}_m,$$

because the $\overline{L}$-module $\wedge^n \overline{V}_1$ is the only summand on the right side which is equivalent to $\overline{V}_1$ by 7) (note that $\overline{V}_i \cong V_i$ as $\overline{L}$-modules for every i). We consider the map $\tau(v_1, \cdots, v_n) := [\mu(v_1), \cdots, \mu(v_n)] - \mu[v_1, \cdots, v_n]$. Since $\pi\tau(v_1, \cdots, v_n)) = 0$ for all $v_i \in \overline{V}$, $i \in \underline{n}$, τ is n-linear map from $\times^n \overline{V}$ to I. It is clear that τ is an alternating map. Moreover, since μ is an $\overline{L}$-module morphism, we have for all $X \in \overline{L}$ and all $v_i \in \overline{V}$, $i \in \underline{n}$:

$$\begin{aligned}
& X.(\tau(v_1, \cdots, v_n)) \\
&= X.([\mu(v_1), \cdots, \mu(v_n)]) - X.(\mu([v_1, \cdots, v_n])) \\
&= \sum_{i=1}^{n} [\mu(v_1), \cdots, \mu(v_{i-1}), \mu(X.v_i), \mu(v_{i+1}), \cdots, \mu(v_n)] \\
&\quad - \sum_{i=1}^{n} \mu[v_1, \cdots, v_{i-1}, X.v_i, v_{i+1}, \cdots, v_n] \\
&= \sum_{i=1}^{n} \tau(v_1, \cdots, v_{i-1}, X.v_i, v_{i+1}, \cdots, v_n).
\end{aligned}$$

Therefore τ induces an $\overline{L}$-module morphism $\tilde{\tau} : \wedge^n \overline{V} \to I$.

Now let $\nu : \overline{V} \to I$ be a nonzero $\overline{L}$-module morphism. Consider the map $\nu_1 : \times^n \overline{V} \to I$ defined by

$$\begin{aligned}
\nu_1(v_1, \cdots, v_n) \;:=\; & \nu[v_1, \cdots, v_n] \\
& - \sum_{i=1}^{n} [\mu(v_1), \cdots, \mu(v_{i-1}), \nu(v_i), \mu(v_{i+1}), \cdots, \mu(v_n)].
\end{aligned}$$

One can prove that ν_1 is alternating and

$$X.(\nu_1(v_1, \cdots, v_n)) = \sum_{i=1}^{n} \nu_1(v_1, \cdots, v_{i-1}, X.v_i, v_{i+1}, \cdots, v_n).$$

Therefore ν_1 induces an $\overline{L}$-module morphism $\tilde{\nu}_1 : \wedge^n \overline{V} \to I$. We assume that $\nu_1 \neq 0$ (in case $\nu_1 = 0$ we take $\beta\mu$ instead of μ where $\beta^{n-1} \neq 1$). Let $\alpha \in K$ such that $\tilde{\tau} = \alpha \tilde{\nu}_1$, i.e. for all $v_i \in \overline{V}$, $i \in \underline{n}$,

$$\begin{aligned}
& [\mu(v_1), \cdots, \mu(v_n)] - \mu[v_1, \cdots, v_n] \\
&= \alpha\nu[v_1, \cdots, v_n] - \alpha \sum_{i=1}^{n} [\mu(v_1), \cdots, \mu(v_{i-1}), \nu(v_i), \mu(v_{i+1}), \cdots, \mu(v_n)].
\end{aligned}$$