root α of L. Since α_0 is maximal, it follows that $\alpha - \beta = \alpha_0 - (\alpha_0 + \beta - \alpha) \succ 0$. But α is simple, hence $\beta = \alpha$. Finally we get $\lambda - \sigma_0\lambda = \alpha_0 + \alpha$. □

In Proposition A13 all pairs (L, V) with (3.3) are determined. We list them here again for convenience.

L	A_1	A_3	$B_l, l \geq 2$	B_3	$D_l, l \geq 4$	D_4	G_2
λ	$2\lambda_1$	λ_2	λ_1	λ_3	λ_1	λ_3, λ_4	λ_1

We shall go through the list and discuss the cases respectively.

Case 1: $L \cong B_l$, $l \geq 2$ or D_l, $l \geq 4$, $\lambda = \lambda_1$.

In this case the L-module V is the natural module K^d $(= V(\lambda_1))$ of the orthogonal Lie algebra $so(d, K)$, where $d \geq 5$, $d \neq 6$. By Table 2 the adjoint module of $so(d, K)$ has the maximal weight λ_2 if $d \geq 6$ respectively $2\lambda_2$ if $d = 5$. By Table 3, if ad: $\wedge^n K^d \to so(d, K)$ is a nonzero $so(d, K)$-module morphism for some n, then $n = 2$ or $n = d - 2$. In the following we exclude the case $n = 2$.

There is up to scalar only one nonzero $so(d, K)$-module morphism τ from $\wedge^2 K^d$ to $so(d, K)$ (see Table 3) and it can be constructed as follows. Let b be the nondegenerate symmetric bilinear form on K^d which defines $so(d, K)$. Let $u, v \in V(\lambda_3)$ be fixed. For the linear form $L \to K : x \to b(x.u, v)$ there exists a unique $\tau(u, v) \in so(d, K)$ such that $\mathrm{Kill}(x, \tau(u, v)) = b(x.u, v)$. Obviously $\tau \neq 0$ and for all $x \in so(d, K)$ we have

$$\begin{aligned}
\mathrm{Kill}(x, \tau(v, u)) &= b(x.v, u) \\
&= -b(v, x.u) \\
&= -b(x.u, v) \\
&= -\mathrm{Kill}(x, \tau(u, v)),
\end{aligned}$$

which implies $\tau(v, u) = -\tau(u, v)$. Therefore τ induces a linear map from $\wedge^2 K^d$ to $so(d, K)$ and we write $\tau(u \wedge v)$ instead of $\tau(u, v)$. Moreover for all $x, y \in so(d, K)$,

$$\begin{aligned}
&\mathrm{Kill}(x, [y, \tau(u \wedge v)]) \\
&= b([x, y], \tau(u \wedge v)) \\
&= b([x, y].u, v) \\
&= b(x.y.u, v) - b(y.x.u, v) \\
&= \mathrm{Kill}(x, \tau(y.u \wedge v)) + b(x.u, y.v) \\
&= \mathrm{Kill}(x, \tau(y.u \wedge v)) + \mathrm{Kill}(x, \tau(u \wedge y.v)).
\end{aligned}$$