which implies that L_1 is an ideal of L_0. Let L_2 be the ideal of L_0 with $L_0 = L_1 \oplus L_2$. Let N be the set of nilpotent endomorphisms of V belonging to R. Then N is an ideal of L (cf. [7] p. 45 or [13] p. 257). Since L is an algebraic Lie algebra (recall that an algebraic Lie algebra is the Lie algebra of an algebraic group, cf. [2] and [8]), R can be represented as the direct sum of N and an abelian algebra A whose elements are semisimple endomorphisms and commute with elements of L_0 (cf. [2]). In one word, we have the following decompositions:

$$L = R + L_0, \quad R = N + A, \quad L_0 = L_1 + L_2$$

with $[A, L_0] = \{0\}$. Set $L' := A + L_0$. Then V is a completely reducible L'-module. Since I is invariant under all derivations (see Remark 2.1), it is an L'-submodule of V. Let V_0 be a complement of I in V: $V = I \oplus V_0$. Set $L'' := A + L_1$. Then $L'' \subseteq L'$ and $L'' \subseteq Ker\gamma$.

In the following we discuss two cases.

Case 1: $L'' \neq \{0\}$

Let V_1 denote the null space of L''. Then $V_1 = \{v \in V \mid D(v) = 0, \forall D \in L''\}$. We see easily that V_1 is a subalgebra of V. If V_1 agrees with V_0, then it is a Levi subalgebra of V because the sum $V = I + V_0$ is direct.

We are going to show that $V_1 = V_0$:

The inclusion $V_0 \subseteq V_1$ results from the fact that $L''(V_0) \subseteq L'(V_0) \subseteq V_0$ and $L''(V_0) \subseteq Ker\gamma(V_0) \subseteq I$, i.e. $L''(V_0) \subseteq V_0 \cap I = \{0\}$. To get the equality of V_1 and V_0 we put $I_0 := I \cap V_1$. Then V_1 is the vector space direct sum of I_0 and V_0. In fact, let $x \in V_1$ and $x = y + z$, $y \in I$, $z \in V_0$. Since $y = x - z \in V_1$, we get $y \in I_0$. It remains to show $I_0 = \{0\}$. For this purpose we prove that I_0 is invariant under all derivations. This is equivalent to the following two inclusions: $L_2(I_0) \subseteq I_0$ and $N(I_0) \subseteq I_0$ because $L'(I_0) = \{0\} \subseteq I_0$. We first show $L_2(I_0) \subseteq I_0$: Since $[L'', L_2] = \{0\}$, we get $L_2(V_1) \subseteq V_1$, hence $L_2(I_0) \subseteq L_2(V_1) \subseteq V_1$. We also have that $L_2(I_0) \subseteq I$ because I is invariant under L, in particular, under L_2. Therefore we get $L_2(I_0) \subseteq I_0$. We now show $N(I_0) \subseteq I_0$: Since $N(I)$ is an ideal of I (see Proposition 2.6), $N(I) = I$ or $\{0\}$. Since the elements of N are nilpotent endomorphisms of V, $N(I)$ is properly included in I by Engel's Theorem. By assumption, I is minimal, $N(I) = \{0\}$ follows. In particular, $N(I_0) = \{0\}$. Consequently I_0 is invariant under all elements of L which implies that I_0 is an ideal of V. Since I is a minimal ideal of V, $I_0 = \{0\}$ or I If $I_0 = I$, we get together with the first inclusion that $V_1 = V_0$ which implies that $L''(V) = \{0\}$, which is a contradiction to the assumption that $L'' \neq \{0\}$. Therefore $I_0 = \{0\}$ and $V_1 = V_0$ follows.

Case 2: $L'' = \{0\}$