V over a field of characteristic 0 is semisimple if and only if it is a direct sum of simple ideals (see Theorem 2.7). In order to prove this we will show that the Lie algebra $\mathrm{Der}(V)$ of derivations of V is semisimple. Then an application of Weyl's theorem on complete reducibility yields the result.

In the third chapter we classify the finite dimensional simple n-Lie algebras over an algebraically closed field K of characteristic 0 and over the real numbers. As in the case of 3-Lie algebras studied in Faulkner [3] we shall show that for $n > 2$ all finite dimensional simple n-Lie algebras over K are isomorphic to each other (see Theorem 3.9). In the proof of this result we will heavily use the representation theory of finite dimensional semisimple Lie algebras. In particular, we need some informations about the decomposition of the n-fold wedge product of an irreducible module of a semisimple Lie algebra.

It turns out that every finite dimensional simple n-Lie algebra over K is of dimension $n+1$. So we get a realization by the multiplication table $(*)$ mentioned above, where all coefficients α_i may be chosen to be 1. Another realization can be given as follows. It generalizes the vector product on K^3 to K^{n+1} (see Example 1.1.1). Let b be a nondegenerate symmetric bilinear form and f be a nonzero determinant form on K^{n+1}. For $v_1, \cdots, v_n \in K^{n+1}$ let $[v_1, \cdots, v_n]$ be the unique element in K^{n+1} such that for all $x \in K^{n+1}$ the identity

$$b([v_1, \cdots, v_n], x) = f(v_1, \cdots, v_n, x)$$

holds. Then K^{n+1} is an n-Lie algebra with product $[v_1, \cdots, v_n]$.

Every real simple n-Lie algebra is isomorphic to an n-Lie algebra $(\mathbb{R}^{n+1}, b, f)$ or a realification of a complex simple n-Lie algebra.

In the fourth chapter we will prove an analog of Levi decompositions of finite dimensional Lie algebras for finite dimensional n-Lie algebras. The result here is that each finite dimensional n-Lie algebra over an algebraically closed field of characteristic 0 has a semisimple subalgebra such that the given n-Lie algebra is the direct sum of its radical and this subalgebra (see Theorem 5.1). Note that one has proved the existence of a Levi decomposition for a Lie algebra by means of cohomology. Unfortunately, we do not have a concept of cohomology for n-Lie algebras. But our results for the structure of semisimple n-Lie algebras provide us with an alternative way to construct a Levi decomposition.

We will begin our study in the first two chapters by introducing some basic concepts for n-Lie algebras. Moreover, we discuss some examples and describe the $(n+1)$-dimensional simple n-Lie algebras. After reviewing the definitions of k-solvability and k-semisimplicity of Kasymov [10] we study the structure of finite dimensional semisimple respectively reductive n-Lie algebras. A reductive n-Lie algebra is by definition an n-Lie algebra whose centre agrees with its radical. We shall show that a finite dimensional n-Lie algebra over a field of characteristic 0 is