Now we have $L = R + L_0$, $R = Ker\gamma = N$ and $L_0 = L' = L_2$. Clearly π induces a Lie algebra isomorphism from L_2 onto $\overline{L}$. Its inverse map will be denoted by σ. Then we can regard V as an $\overline{L}$-module as follows: $X.v = \sigma(X)(v)$.

1) $N(I) = \{0\}$. This has been shown above.

2) $[I, I, V, \cdots, V] = \{0\}$. Since $\mathrm{ad}(I, V, \cdots, V) \subseteq Ker\gamma$, it follows from the assumption that $\mathrm{ad}(I, V, \cdots, V) \subseteq N$. By 1), $[I, I, V, \cdots, V] = \{0\}$.

3) *I is an irreducible $\overline{L}$-module.* If J is a proper L'-submodule of I, then because of $N(J) \subseteq N(I) = \{0\}$, J must be an ideal of V contradicting the minimality of I. Therefore I is an irreducible L'-module which implies the assertion.

4) *The complementary $\overline{L}$-submodule V_0 is equivalent to the $\overline{L}$-module $\overline{V}$.* In fact, let π_1 denotes the restriction of π on V_0. Then we have for all $X \in \overline{L}$ and all $v \in V_0$:

$$\pi_1(X.v) = \pi(\sigma(X)(v)) = \gamma(\sigma(X))(\pi(v)) = X(\pi(v)).$$

That is, $\pi_1 : V_0 \to \overline{V}$ is an $\overline{L}$-module isomorphism. Let μ be the inverse of π_1.

5) *Set $V_i := \mu(\overline{V}_i)$, $i \in \underline{m}$. Then V_i is an $\overline{L}$-submodule of V_0 that is equivalent to $\overline{V}_i$ and $V_0 = \oplus_{i=1}^m V_i$. Moreover, $\overline{L}_i(V_j) = \{0\}$ for $i \neq j$ and V_i is the natural $\overline{L}_i$-module (or the natural $so(n+1, K)$-module $V(\lambda_1)$).*

6) *The $\overline{L}$-module $[V_0, \cdots, V_0]$ contains a submodule that is equivalent to V_0, hence it contains an $\overline{L}$-submodule equivalent to V_i.* This is evident because we have

$$\pi[V_0, \cdots, V_0] = [\pi(V_0), \cdots, \pi(V_0)] = [\overline{V}, \cdots, \overline{V}] = \overline{V},$$

7) *We have the following decomposition:*

$$[V_0, \cdots, V_0] = \sum_{\substack{0 \leq n_1, \cdots, n_m \leq n \\ n_1 + \cdots + n_m = n}} V_{(n_1, \cdots, n_m)},$$

where

$$V_{(n_1, \cdots, n_m)} := [\underbrace{V_1, \cdots, V_1}_{n_1}, \cdots, \underbrace{V_m, \cdots, V_m}_{n_m}].$$

Clearly, each $V_{(n_1, \cdots, n_m)}$ is an $\overline{L}$-submodule of V and is equivalent to a submodule of $V^{(n_1, \cdots, n_m)} := \wedge^{n_1} V_1 \otimes \wedge^{n_2} V_2 \otimes \cdots \otimes \wedge^{n_m} V_m$. Furthermore, any two distinct $\overline{L}$-modules $V^{(n_1, \cdots, n_m)}$ and $V^{(n_1', \cdots, n_m')}$ contain no nonzero equivalent submodules. We show the last assertion by showing that if U resp. U' is a nonzero submdule of $V^{(n_1, \cdots, n_m)}$ resp. $V^{(n_1', \cdots, n_m')}$ with $U \cong U'$, then $(n_1, \cdots, n_m) = (n_1', \cdots, n_m')$. Let

$$U \cong U_1 \otimes \cdots \otimes U_m \text{ and } U' \cong U_1' \otimes \cdots \otimes U_m',$$