Theorem 2.7: *Let K be of characteristic 0. An n-Lie algebra V is semisimple if and only if V is a direct sum of simple ideals: $V = \oplus_{i=1}^m V_i$, $m \in \mathbb{N}$. Moreover, $Der(V)$ is semisimple and each derivation of V is inner.*

Proof: If V is a direct sum of simple ideals, then V is semisimple by Theorem 2.5.

Let now V be semisimple. We prove that V is a direct sum of simple ideals. Let R be the radical of L' (recall $L' = Der(V)$ and $L = Inder(V)$) and $M := [R, L']$. According to Theorem 12.38 in [13] M is included in the radical of the associative subalgebra of End(V) generated by L', hence $M^{s+1} = \{0\}$ for some $s \in \mathbb{N}$. By Proposition 2.6, $(M(V))^{(s,n)} \subseteq M^{s+1}(V) = \{0\}$. This says that $M(V)$ is a solvable ideal of V, hence by the assumption, $M(V) = \{0\}$ which implies $M = \{0\}$, that is, R coincides with the centre of L'. Therefore L' is reductive.

Let Z be the centre of L'. Since V is semisimple, $C(V) = \{0\}$. Applying Lemma 1.1.4 to elements of Z we obtain $Z = \{0\}$. This means that L' is a semisimple Lie algebra. Let L_1 be the ideal of L' with $L' = L \oplus L_1$. Once again by Lemma 1.1.4, $L_1 = \{0\}$ because of $[L_1, L] = \{0\}$. Therefore $L' = L$.

Now the L-module V is completely reducible. Suppose that $V = \oplus_{i=1}^m V_i$, where V_i are irreducible L-modules. This is at the same time a direct sum of ideals of V. If V_i is one dimensional, then $V_i \subseteq C(V)$. But $C(V) = \{0\}$ by the assumption. Therefore V_i is nontrivial. Since V_i is an irreducible L-module, V_i contains no ideal of V. But each ideal of V_i is also an ideal of V, hence V_i is simple as a subalgebra of V. This completes the proof of Theorem 2.7. □

Corollary 2.8: *Let K be of characteristic 0. If V is a semisimple n-Lie algebra, then every simple ideal of V coincides with one of the V_i in Theorem 2.7 and any ideal is a direct sum of certain simple ideals.*

Proof: Let $V = \oplus V_i$ be the decomposition of V into simple ideals. If I is a simple ideal of V, then $[I, V, \cdots, V]$ is an ideal of V, nonzero because the centre of V is zero. This forces $[I, V, \cdots, V] = I$, since I is simple. On the other hand, $[I, V, \cdots, V] = \oplus_{i=1}^m [I, V_i, \cdots, V_i]$, so all but one summand must be 0. Say $I = [I, V_i, \cdots, V_i]$. Then $I \subseteq V_i$, and $I = V_i$ because V_i is simple. Therefore V_i are all simple ideals of V, $i \in \underline{m}$.

Next we prove that each ideal of V is the sum of some V_i's. Let I be an arbitrary ideal of V. Then I is an invariant subspace of the L-module V. But L is semisimple, I is a direct sum of some irreducible L-submodule of V, in other words, I is a direct sum of simple ideals of V. □

In the following we are concerned with the connection between the radical of the Lie algebra of derivations of V and the radical of V. For this purpose we make