Chapter 2

Solvability of n-Lie algebras

In this chapter we are going to study the structure of n-Lie algebras. We shall introduce the concepts of k-solvability and k-semisimplicity for n-Lie algebras. Then we give a characterization of n-semisimple n Lie algebras. The reductive n-Lie algebras will also be studied. Throughout K will be a field of characteristic different from 2.

Let us recall the definition of Kasymov [10] for solvability.

Let V be an n-Lie algebra over K. For a given ideal I of V and a given $k \in \underline{n}$ we define inductively a sequence of ideals $I^{(s,k)}$, $s \in \mathbb{N}_0$, of V:

$$I^{(0,k)} := I, \quad I^{(s+1,k)} := [\underbrace{I^{(s,k)}, \cdots, I^{(s,k)}}_{k}, V, \cdots, V].$$

Clearly we have $I^{(s+1,k)} \subseteq I^{(s,k)}$.

Definition of k-solvability:

1) *An ideal I of V is called a k-solvable ideal of V if for some $s \in \mathbb{N}$: $I^{(s,k)} = \{0\}$.*

2) *V is called a k-solvable n-Lie algebra if V is k-solvable as an ideal of itself.*

Let V be an n-Lie algebra and I an ideal of V. According to the definition, if I is a k-solvable ideal of V, then I is a k-solvable subalgebra of V. We shall see in Example 2.1 below that the converse is not true except in case $k = n$. In the following we consider some extrem cases. If $k = n$, the definition agrees with that of Filippov [5]; if $n = 2$, then the 2-solvability is the same as the solvability for Lie algebras. To understand the 1-solvability we define the so-called upper central series. Let $C_0(V) := \{0\}$, $C_{s+1} = \{v \in V \mid [V, \cdots, V, v] \subseteq C_s\}$ for $s \geq 0$. Clearly $C_s(V)$ has the following properties:

1) *$C_1(V)$ is the centre $C(V)$ of V.*

2) *For each $s \in \mathbb{N}_0$, $C_s(V)$ is a 1-solvable ideal of V.*

3) *For each $s \in \mathbb{N}_0$, $C_s(V) \subseteq C_{s+1}(V)$.*