

the following consideration.

Let V_i , $i = 1, 2$, be two n -Lie algebras over an arbitrary field K . Let L_i denote the Lie algebra of derivations of V_i , $i = 1, 2$. Suppose that $\pi : V_1 \rightarrow V_2$ is a surjective homomorphism of n -Lie algebras such that the kernel of π is invariant under L_1 . Define

$$\gamma : L_1 \rightarrow L_2 \quad \gamma(D)(\pi(v)) := \pi(D(v)), \quad D \in L_1, v \in V. \quad (2.1)$$

Evidently $\gamma(D)$ is an endomorphism of V_2 . It is even an element of L_2 because we have for all $v_i \in V_1$, $i \in \underline{n}$:

$$\begin{aligned} & \gamma(D)[\pi(v_1), \dots, \pi(v_n)] \\ &= \gamma(D)(\pi[v_1, \dots, v_n]) \\ &= \pi(D[v_1, \dots, v_n]) \\ &= \pi\left(\sum_{i=1}^n [v_1, \dots, v_{i-1}, D(v_i), v_{i+1}, \dots, v_n]\right) \\ &= \sum_{i=1}^n [\pi(v_1), \dots, \pi(v_{i-1}), \pi(D(v_i)), \pi(v_{i+1}), \dots, \pi(v_n)] \\ &= \sum_{i=1}^n [\pi(v_1), \dots, \pi(v_{i-1}), \gamma(D)(\pi(v_i)), \pi(v_{i+1}), \dots, \pi(v_n)]. \end{aligned}$$

Let $D_1, D_2 \in L_1$ and $v \in V_1$. Then

$$\begin{aligned} \gamma([D_1, D_2])(\pi(v)) &= \pi([D_1, D_2]v) \\ &= \pi(D_1 D_2(v) - D_2 D_1(v)) \\ &= \gamma(D_1)(\pi(D_2(v))) - \gamma(D_2)(\pi(D_1(v))) \\ &= \gamma(D_1)\gamma(D_2)(\pi(v)) - \gamma(D_2)\gamma(D_1)(\pi(v)) \\ &= [\gamma(D_1), \gamma(D_2)](\pi(v)). \end{aligned}$$

Hence γ is a Lie algebra homomorphism from L_1 to L_2 . Moreover, we have for any $\text{ad}_1(u_1, \dots, u_{n-1}) \in L_1$:

$$\begin{aligned} \gamma(\text{ad}_1(u_1, \dots, u_{n-1}))(\pi(v)) &= \pi([u_1, \dots, u_{n-1}, v]) \\ &= [\pi(u_1), \dots, \pi(u_{n-1}), \pi(v)] \\ &= \text{ad}_2(\pi(u_1), \dots, \pi(u_{n-1}))(\pi(v)), \end{aligned}$$

that is, $\gamma(\text{ad}_1(u_1, \dots, u_{n-1})) = \text{ad}_2(\pi(u_1), \dots, \pi(u_{n-1}))$. Therefore

$$\gamma(\text{Inder}(V_1)) = \text{Inder}(V_2). \quad (2.2)$$

Now let V be an n -Lie algebra over a field K . Since the canonical homomorphism $\pi : V \rightarrow V/\text{Rad}(V)$ is surjective and its kernel $\text{Rad}(V)$ is invariant under all