for some simple root α of L, then the pair (L, λ) is one of following:

L	λ
A_1	$2\lambda_1$
A_3	λ_2
$B_l, l \geq 2$	λ_1
B_3	λ_3
$D_l, l \geq 4$	λ_1
D_4	λ_3, λ_4
G_2	λ_1

where l denotes the rank of L.

Proof: From (A.2) we get

$$\sigma_0(\alpha_0 + \alpha) + (\alpha_0 + \alpha) = \sigma_0(\lambda - \sigma_0\lambda) + (\lambda - \sigma_0\lambda) = 0.$$

But $\sigma_0\alpha_0 = -\alpha_0$, hence the simple root α satisfies

$$\sigma_0\alpha + \alpha = 0 \tag{A.3}$$

If $l = 1$, then L possesses only one positive root α_0. Since each L-module is self-contragredient, that is, $\sigma_0\lambda = -\lambda$ for all $\lambda \in \Lambda^+$, we get from $2\lambda = 2\alpha_0$ that $\lambda = \alpha_0$, in other words, V is the adjoint A_1-module. If $l = 2$, then L is of type A_2, C_2 or G_2. Let $\Delta = \{\alpha_1, \alpha_2\}$. If $L \cong A_2$, then $\sigma_0\alpha_1 = -\alpha_2$, $\sigma_0\alpha_2 = -\alpha_1$. Thus there is no simple root of A_2 satisfying (A.3). If $L \cong G_2$, then $\alpha_0 = \lambda_2$ and $\sigma_0 = -1$. Now (A.2) becomes $2\lambda = \lambda_2 + \alpha$. The simple roots of G_2 are $2\lambda_1 - \lambda_2$ and $-3\lambda_1 + 2\lambda_2$. Thus $2\lambda = 2\lambda_1$ or $-3\lambda_1 + 3\lambda_2$. Since λ is dominant, $\lambda = \lambda_1$. The maximal root of C_2 is $2\lambda_1$ and $\sigma_0 = -1$. If $\alpha = \alpha_1$, then $\alpha_0 + \alpha$ is not dominant since $\langle \alpha_0 + \alpha, \alpha_2 \rangle = \langle 2\lambda_1 + \alpha_1, \alpha_2 \rangle = -1 < 0$. This contradicts to the condiction that $\alpha_0 + \alpha$ is dominant. If $\alpha = \alpha_2$. $2\lambda = 2\lambda_1 + \alpha_2 = 2\lambda_1 + (-2\lambda_1 + 2\lambda_2) = 2\lambda_2$, thus $\lambda = \lambda_2$. It is well-known that the C_2-module $V(\lambda_2)$ is isomorphic to the B_2-module $V(\lambda_1)$. Now let $l \geq 3$. If α' is a neighbour of α in the Dynkindiagram, then

$$\langle \alpha_0, \alpha' \rangle = \langle \lambda - \sigma_0\lambda - \alpha, \alpha' \rangle > 0. \tag{A.4}$$

Hence if the simple root α possesses two neighbours in the Dynkindiagram, then there are at least two positive coefficients in the expression of α_0 relative to the fundamental weights. By Table 2, L is of type A_l. Since $\sigma_0\alpha_i = -\alpha_{l+1-i}$ for all $i \in \underline{l}$, $l > 3$ must be odd by (A.3) and $\alpha = \alpha_{\frac{l+1}{2}}$. If $l > 3$, then $\alpha_0 + \alpha_{\frac{l+1}{2}}$ is not dominant, hence $l = 3$ and $\alpha = \alpha_2$. This gives rise to $L \cong A_3$ and $\lambda = \lambda_2$.

Now assume that α is an end point and α_i is the unique neighbour of α in the Dynkindiagram (notice that α_i is no end point). Then $\langle \alpha_0, \alpha_i \rangle > 0$, i.e. the