Thus $[y, \tau(u \wedge v)] = \tau(y.u \wedge v) + \tau(u \wedge y.v)$, i.e., τ is an $so(d, K)$-module morphism from $\wedge^2 K^d$ to $so(d, K)$. Since $so(d, K)$ $(d \geq 5)$ is irreducible as a module of itself, τ is surjective.

If $\tau(u, v).u = 0$ for all $u, v \in K^d$, then $\mathrm{Kill}(\tau(u \wedge v), \tau(u \wedge v)) = b(\tau(u \wedge v).u, v) = 0$. This means that the Killing form of L is skew-symmetric. Therefore the map $(u, v, w) \to \tau(u \wedge v).w$ is not alternating and $(so(d, K), K^d, \tau)$ is not a good triple.

It remains to check whether $(so(d, K), K^d, \mathrm{ad})$ is a good triple where ad is the (up to scalar) unique $so(d, K)$-module morphism from $\wedge^{d-2} K^d$ to $so(d, K)$. But we need not to do this because Theorem 1.2.4 shows that the vector product (K^d, b, f) is a simple $(d-1)$-Lie algebra with $so(d, K)$ as its derivation algebra and it is the only simple $(d-1)$-Lie algebra of dimension d up to isomorphism.

Case 2: $L \cong A_1$, $\lambda = 2\lambda_1$.

Since the $sl(2, K)$-module V with the maximal weight $2\lambda_1$ is 3-dimensional (notice that V is just the adjoint module of $sl(2, K)$), L and V give us no good triple.

Case 3: $L \cong A_3$, $\lambda = \lambda_2$.

The 6-dimensional $sl(4, K)$-module $V(\lambda_2)$ is self-contragredient, that is, there is an $sl(4, K)$-invariant nondegenerate bilinear form b on $V(\lambda_2)$ (b is unique up to scalar), since $\sigma_0 \lambda = -\lambda_2$ (see proof of Theorem A13). Then the representation of $sl(4, K)$ in $V(\lambda_2)$ gives a Lie algebra isomorphism $\rho : sl(4, K) \to L_1$, where L_1 is the set of the endomorphisms τ of $V(\lambda_2)$ with $b(\tau u, v) + b(u, \tau v) = 0$ for all $u, v \in V(\lambda_2)$, since $\dim sl(4, K) = \dim L_1 = 15$ and ρ is faithful. We know that b is either symmetric or skew-symmetric (cf. [14]). If the second case is true, then $sl(4, K)$ is isomorphic to $sp(6, K)$ which is impossible because $\dim sp(6, K) = 21$. Therefore b is symmetric. It follows that $sl(4, K) \cong so(6, K)$ and the $sl(4, K)$-module $V(\lambda_2)$ can be regarded as the natural $so(6, K)$-module. By equivalence of the good triples it suffices to consider the good triples based on $so(6, K)$ and K^6. As in Case 1 we can conclude that there is only one n-Lie algebra structure on K^6 up to isomorphism and its derivation algebra is isomorphic to $so(6, K)$.

Case 4: $L \cong B_3$, $\lambda = \lambda_3$.

V is the 8-dimensional spin representation of $so(7, K)$. By the computer program " Lie " by Arjeh M. Cohen, Bert Lisser, Bart de Smit and Ron Sommeling we have the following decompositions (all decompositions which appear later are also based on this programm) where $\wedge^m V$ ($\vee^m V$) denotes the alternating (symmetric) part of the tensor product.

$$\wedge^2 V(\lambda_3) \cong V(\lambda_1) \oplus V(\lambda_2) \tag{3.8}$$