derivations of V (see Remark 2.1), we can define (as (2.1)) a homomorphism $\gamma : Der(V) \to Der(V/Rad(V))$ with

$$Ker\,\gamma = \{D \in Der(V) \mid D(V) \subseteq Rad(V)\}. \tag{2.3}$$

By Theorem 2.4, $V/Rad(V)$ is semisimple. If $\mathrm{char}K = 0$, then $Der(V/Rad(V))$ agrees with $Inder(V/Rad(V))$ and is semisimple by Theorem 2.7. But by (2.2), $\gamma(Inder(V)) = Inder(V/Rad(V))$, it follows that γ is a surjective Lie algebra homomorphism and Rad (Der(V)) $\subseteq Ker\gamma$. So we have proved

Theorem 2.9: *Let K be of characteristic 0. Let V be an n-Lie algebra over K. Let $\pi : V \to V/Rad(V)$ be the canonical homomorphism. Define a map $\gamma : Der(V) \to Inder(V/Rad(V))$ by $\gamma(D)(\pi(v)) := \pi(D(v))$. Then γ is a surjective Lie algebra homomorphism with the kernel given in (2.3). Moreover $Rad(Der(V)) \subseteq Ker\gamma$ and $Rad(Der(V))(V) \subseteq Rad(V)$.*

We now generalize the concept of reductive Lie algebras.

Definition: *An n-Lie algebra V is called reductive if $Rad(V) = C(V)$.*

Example 2.3: Let $n \geq 3$ and $V = (K^{n+1}, b, f)$ be the n-Lie algebra with the vector product. For given $x \in V$ we consider the $(n-1)$-Lie algebra $V(x)$ on K^{n+1} with the product $[v_1, \cdots, v_{n-1}]_x := [v_1, \cdots, v_{n-1}, x]$ (see Remark 1.1.1). Obviously Kx is an ideal of $V(x)$ included in the centre of $V(x)$. Set $V_0 := \{v \in V \mid b(v, x) = 0\}$. Now

$$\begin{aligned} & b([V(x), \cdots, V(x)]_x, x) \\ &= b([V, \cdots, V, x], x) \\ &= f(V, \cdots, V, x, x) = \{0\}. \end{aligned}$$

Thus $[V(x), \cdots, V(x)]_x \subseteq V_0$, which implies that V_0 is an ideal of $V(x)$.

Case 1: $b(x, x) = 0$.

By the assumption $x \in V_0$. Let W be a subspace of V_0 such that the sum $V_0 = Kx + W$ is direct. Then $V_0^{(1,n-1)} = [V_0, \cdots, V_0]_x = [W, \cdots, W]_x$ and has dimension 1 (notice $\dim W = n-1$). Therefore V_0 is solvable. As a consequence $V(x)$ is solvable because $[V(x), \cdots, V(x)]_x \subseteq V_0$.

Case 2: $b(x, x) \neq 0$.

Since $V = Kx + V_0$ (direct sum of vector spaces), the $(n-1)$-Lie algebra $V(x)$ is the direct sum of its ideals Kx and V_0. We show that V_0 is a simple $(n-1)$-Lie algebra.