

x_i is given by

$$x_i = (-1)^{n+i} \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{D_i a_1} & \widehat{D_i a_2} & \cdots & \widehat{D_i a_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_{n-1} \end{vmatrix}.$$

If $\sum_{i=1}^n D_i x_i$ is 0, then G.J.I. results from identity (1.8). Thus it suffices to show that $\sum_{i=1}^n D x_i = 0$. In fact,

$$\begin{aligned} \sum_{i=1}^n D_i x_i &= \sum_{i=1}^n \sum_{j=1, j \neq i}^n (-1)^{n+i} \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{D_i a_1} & \widehat{D_i a_2} & \cdots & \widehat{D_i a_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ D_i D_j a_1 & D_i D_j a_2 & \cdots & D_i D_j a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_{n-1} \end{vmatrix} \\ &= \sum_{1 \leq i < j \leq n} (-1)^{n+i} \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{D_i a_1} & \widehat{D_i a_2} & \cdots & \widehat{D_i a_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ D_i D_j a_1 & D_i D_j a_2 & \cdots & D_i D_j a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_{n-1} \end{vmatrix} \\ &\quad + \sum_{1 \leq i < j \leq n} (-1)^{n+j} \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ D_i D_j a_1 & D_i D_j a_2 & \cdots & D_i D_j a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{D_j a_1} & \widehat{D_j a_2} & \cdots & \widehat{D_j a_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_{n-1} \end{vmatrix}. \end{aligned}$$

By moving the i -th row to the $(j-1)$ -th row in the determinant in the second sum without changing the ordering of the remaining rows, we get the following expression:

$$(-1)^{j-i-1} \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ \widehat{D_i a_1} & \widehat{D_i a_2} & \cdots & \widehat{D_i a_{n-1}} \\ \cdots & \cdots & \cdots & \cdots \\ D_i D_j a_1 & D_i D_j a_2 & \cdots & D_i D_j a_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_{n-1} \end{vmatrix}.$$