Proof: $\widetilde{V}_{\mathbb{R}}$ is semisimple, since if I is a solvable ideal of $\widetilde{V}_{\mathbb{R}}$, then $I + i\,I$ is a solvable ideal of $\widetilde{V}$. Let I be an arbitrary ideal of $\widetilde{V}_{\mathbb{R}}$. We show that I is also an ideal of $\widetilde{V}$. Now for all $v_j \in I$, $j \in \underline{n}$, we have by definition of scalar multiplication in $\widetilde{V}$ that

$$i[v_1, \cdots, v_n] = [iv_1, v_2, \cdots, v_n] \in [\widetilde{V}, v_2, \cdots, v_n] \subseteq I.$$

Thus $iI \subseteq I$, since $[I, \cdots, I] = I$. This means that I is closed relative to the scalar multiplication by complex numbers. Thus I is an ideal of $\widetilde{V}$. But $\widetilde{V}$ is simple, hence $I = \widetilde{V} = \widetilde{V}_{\mathbb{R}}$ or $\{0\}$. □

Proposition 3.11: *The complexification $\widetilde{V}$ of a real semisimple n-Lie algebra V is semisimple.*

Proof: By Theorem 1.1.3 the Lie algebra $L := Inder(V)$ of the derivations of V is semisimple. Thus the complexification $\widetilde{L}$ of L is semisimple in view of Proposition A3 . It turns out that $Inder(\widetilde{V})$ agrees with $\widetilde{L}$, thus $\widetilde{V}$ is reductive by Theorem 2.10.

Let $u + iv$ be an element of the centre of $\widetilde{V}$. From $[u + iv, V, \cdots, V] = \{0\}$ we get $[u, V, \cdots, V] = \{0\}$ and $[v, V, \cdots, V] = \{0\}$, that is, u and v are in the centre of V, hence $u = v = 0$ because V is semisimple. This means that the centre of $\widetilde{V}$ is zero. □

Theorem 3.12: *A real simple n-Lie algebra V is isomorphic to the realification of a simple complex n-Lie algebra or to a real form of a simple complex n-Lie algebra.*

Proof: We consider the complexification $\widetilde{V}$ of V. By Proposition 3.10, $\widetilde{V}$ is semisimple. If $\widetilde{V}$ is simple, then V is a real form for some simple complex n-Lie algebra. Assume that $\widetilde{V}$ is not simple. Let C denote the map on $\widetilde{V}$ with $C(u + i\,v) = u - i\,v$. We show that an ideal I of $\widetilde{V}$ with $C(I) = I$ is either $\{0\}$ or $\widetilde{V}$. The fixed points of C in I form an ideal of V: $I_0 := \{x + C(x) \mid x \in I\}$, thus $I_0 = V$ or $\{0\}$ because V is simple. In the second case we have that $I \subseteq iV$ which is impossible since I is an ideal of $\widetilde{V}$. Thus $V = I_0 \subseteq I$ and $I = \widetilde{V}$.

If I is a simple ideal of $\widetilde{V}$, so is $C(I)$. Therefore $I \cap C(I)$ is either $\{0\}$ or I. If $I \cap C(I) = I$, that is $C(I) = I$, then $I = \widetilde{V}$ as shown above, contradiction. If $I \cap C(I) = \{0\}$, then the sum $J := I \oplus C(I)$ is direct and an ideal of $\widetilde{V}$ satisfying $C(J) = J$, it follows again from above that $\widetilde{V} = I + C(I)$.

As a result of it, $V = \{x + C(x) \mid x \in I\}$. Then the correspondence $x \to x + C(x)$ gives a real isomorphism from I onto V, so V is $\mathbb{R}$-isomorphic to the realification of the simple ideal I of $\widetilde{V}$. □

According to Theorem 3.12, in order to find all real simple n-Lie algebras ($n \geq 3$)