we conclude that $\mathrm{ad}(x_\alpha.v^- \wedge v^- \wedge v_{\mu_1} \wedge \cdots \wedge v_{\mu_{n-3}})$ is different from 0 and has $2\sigma_0\lambda + \alpha + \sum_{i=1}^{n-3}\mu_i$ as its weight. But $\lambda + \sigma_0\lambda + \sum_{i=1}^{n-3}\mu_i = 0$, hence $-\lambda + \sigma_0\lambda + \alpha \in \Phi \cup \{0\}$, or equivalently $\lambda - \sigma_0\lambda - \alpha \in \Phi \cup \{0\}$.

If $\lambda - \sigma_0\lambda - \alpha = 0$, then $\lambda - \sigma_0\lambda$ is a root of L which is impossible by Lemma 3.2. Therefore $\lambda - \sigma_0\lambda - \alpha \in \Phi$. □

Finally we proceed to C3).

To find all good triples we look for the pairs L and V where L is a finite dimensional semisimple Lie algebra and V is a faithful irreducible L-module with the property in Lemma 3.5. For this purpose we discuss two different cases, namely L is simple or not.

At first we assume that L is not simple, say $L = \oplus_{i=1}^{s} L_i$, where L_i are the simple ideals of L and $s \geq 2$. Further let $H_i := H \cap L_i$, $\Phi_i \subseteq \Phi$ the root system of L_i relative to H_i and $\Delta_i = \Delta \cap \Phi_i$ (see Theorem A6 and Remark A1). First, we can assume that $\alpha \in \Delta_1$. Then $\alpha(H_i) = \{0\}$ for all $2 \leq i \leq s$. Let $\beta = \lambda - \sigma_0\lambda - \alpha$. If $\beta \in \Phi_1$, then $\beta(H_i) = \{0\}$ for all $2 \leq i \leq s$. Therefore $(\lambda - \sigma_0\lambda)(H_i) = \{0\}$. Denote by σ_i the element in the Weyl group of L_i with the property: $\sigma_i\Delta_i = -\Delta_i$, and let $\lambda^{(i)}$ be the restriction of λ to H_i. Then $\lambda^{(i)} - \sigma_i\lambda^{(i)} = 0$, which in turn implies $\lambda^{(i)} = 0$ for all $i \geq 2$. But this is impossible because V is faithful (see Corollary A12). Hence $\beta \in \Phi_i$ for some $i \geq 2$, say $i = 2$. The same argument gives $s = 2$, that is, $L = L_1 \oplus L_2$. It follows that $\alpha = \lambda^{(1)} - \sigma_1\lambda^{(1)}$, that is, the simple root α is a sum of two nonzero dominant weights. This is only possible if the rank of L_1 is 1. Thus $L_1 \cong so(3, K)$ and $\lambda^{(1)}$ is the only fundamental dominant weight of L_1.

Assume now that $\alpha \in \Delta_2$. Repeating the above consideration, we can also conclude that $L_2 \cong so(3, K)$ and $\lambda^{(2)}$ is the only fundamental dominant weight L_2. Summarizing the foregoing results we obtain $L \cong so(4, K)$ and $V \cong K^4$.

Can we construct from $so(4, K)$ and K^4 a good triple? Since we have to deal with a 4-dimensional $so(4, K)$-module, this question is the same as to ask whether there exists an $so(4, K)$-module morphism of the form ad: $\wedge^2 K^4 \to so(4, K)$ with G3), or equivalently whether there exists a simple 3-Lie algebra structrue on K^4 such that the Lie algebra of its derivations is isomorphic to $so(4, K)$. Theorem 1.2.4 shows that such a 3-Lie algebra exists and it is unique up to isomorphism. Therefore we have proved

Theorem 3.6: *If V is a finite dimensional simple n-Lie algebra over an algebraically closed field K of characteristic 0 for which the Lie algebra of the derivations is not simple, then $n = 3$. Moreover V is isomorphic to the 3-Lie algebra (K^4, b, f) with the vector product.*

Let us now treat the case that L is simple. Denote by α_0 the maximal root of