3) If $\alpha \in \Phi$, then $\sigma_\alpha(\Phi) = \Phi$.

4) $\langle \alpha, \beta \rangle \in \mathbb{Z}$ for all $\alpha, \beta \in \Phi$.

The elements of Φ are called roots. A base of Φ is a linearly independent subset Δ of Φ such that each β can be written as $\beta = \sum_{\alpha \in \Delta} k_\alpha \alpha$, where k_α are all nonnegative or nonpositive. In the first case β is called a positive root ($\alpha \succ 0$) and in the second case a negative root ($\alpha \prec 0$) with respect to the base Δ. Let Φ^+ (Φ^-) denote the set of positive (resp. negative) roots, then $\Phi = \Phi^+ \cup \Phi^-$. For given $\lambda, \mu \in E$, we write $\lambda \prec \mu$ if $\mu - \lambda$ is a linear combination of elements in Φ^+ with nonnegative coefficients. This gives a half ordering on E.

The subgroup W of $\mathrm{Aut}(E)$ generated by the reflections σ_α, $\alpha \in \Phi$ is the so-called Weyl group of Φ. W acts simply transitively on the bases of Φ. If Δ is a base of Φ, then so is $-\Delta$ and therefore there is a unique element $\sigma_0 \in W$ satisfying $\sigma_0 \Delta = -\Delta$ and $\sigma_0^2 = 1$. For later use we write this fact as

Proposition A5: *For each base Δ of Φ there exists a $\sigma_0 \in W$ with $\sigma_0^2 = 1$, $\sigma_0 \Delta = -\Delta$.*

Define $\Lambda := \{\lambda \in E \mid \langle \lambda, \alpha \rangle \in \mathbb{Z},\ \forall \alpha \in \Delta\}$. A $\lambda \in \Lambda$ is called a weight of Φ. It is dominant if $\langle \lambda, \alpha \rangle \geq 0$ for all $\alpha \in \Delta$. We denote by Λ^+ the set of dominant weights. The elements $\lambda_i \in \Lambda^+$ ($i \in \underline{l}$) with $\langle \lambda_i, \alpha_j \rangle = \delta_{ij}$ for $\alpha_j \in \Delta = \{\alpha_1, \cdots, \alpha_l\}$, are called the fundamental dominant weights. Obviously $\{\lambda_1, \cdots, \lambda_l\}$ is a base of E, and every element $\mu \in E$ possesses the representation $\mu = \sum_{i=1}^{l} \langle \mu, \alpha_i \rangle \lambda_i$.

Assume that L is a semisimple Lie algebra and H a maximal toral subalgebra of L. A toral subalgebra is by definition a subalgebra consisting of semisimple elements. Relative to H we can decompose L into root spaces: $L = H \oplus \oplus_{\alpha \in \Phi} L_\alpha$, where $L_\alpha := \{x \in L \mid [h, x] = \alpha(h)x,\ \forall h \in H\}$ and Φ is the set of $\alpha \in H^*$ for which $L_\alpha \neq \{0\}$. Since all maximal toral subalgebras are conjugate to each other, the above decomposition (Cartan decomposition) is unique up to isomorphim (cf. [7] p. 82).

View H^* as a vector space over $\mathbb{Q}$ and let $E_{\mathbb{Q}}$ be the collection of all $\mu \in H^*$ which are a linear combination of elements in Φ with rational coefficients. Extending the field $\mathbb{Q}$ to $\mathbb{R}$, we obtain a real vector space E. Since the restriction of the Killing form to H is nondegerate, the identity $(\lambda, \mu) := \mathrm{Kill}(t_\lambda, t_\mu)$ gives rise to a nondegenerate symmetric bilinear form on H^*, where t_λ is the unique element in H satisfying $\mathrm{Kill}(t_\lambda, h) = \lambda(h)$ for all $\lambda \in H^*$. The bilinear form $(\cdot, \cdot)$ on H^* induces in turn a positive definite symmetric bilinear form on E. Therewith E becomes a Euclidean space and Φ a root system in E. Φ is called the root system of L relative to H. We know that there is only one semisimple Lie algebra to a given root system up to isomorphism.