

In fact, for all $a_i \in A$, $i \in \underline{n}$,

$$\begin{aligned}
& D[a_1, \dots, a_n] - \sum_{j=1}^n [a_1, \dots, Da_j, \dots, a_n] \\
&= D \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_n \end{vmatrix} \\
&\quad - \sum_{j=1}^n \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 Da_j & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & D_2 Da_j & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & D_n Da_j & \cdots & D_n a_n \end{vmatrix} \\
&= \sum_{j=1}^n \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & DD_1 a_j & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & DD_2 a_j & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & DD_n a_j & \cdots & D_n a_n \end{vmatrix} \\
&\quad - \sum_{j=1}^n \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 Da_j & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & D_2 Da_j & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & D_n Da_j & \cdots & D_n a_n \end{vmatrix} \\
&= - \sum_{j=1}^n \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & [D_1, D]a_j & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & [D_2, D]a_j & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & [D_n, D]a_j & \cdots & D_n a_n \end{vmatrix} \\
&= - \sum_{j=1}^n \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & \sum_{i=1}^n (D_1 x_i)(D_i a_j) & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & \sum_{i=1}^n (D_2 x_i)(D_i a_j) & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & \sum_{i=1}^n (D_n x_i)(D_i a_j) & \cdots & D_n a_n \end{vmatrix} \\
&= - \left(\sum_{i=1}^n D_i x_i \right) \begin{vmatrix} D_1 a_1 & D_1 a_2 & \cdots & D_1 a_n \\ D_2 a_1 & D_2 a_2 & \cdots & D_2 a_n \\ \dots & \dots & \dots & \dots \\ D_n a_1 & D_n a_2 & \cdots & D_n a_n \end{vmatrix} \\
&= - \left(\sum_{i=1}^n D_i x_i \right) \cdot [a_1, \dots, a_n].
\end{aligned}$$

Let us now consider the left multiplications. We claim that (1.8) is true for any left multiplication $D = \text{ad}(a_1, \dots, a_{n-1})$ because we can write $D = \sum_{i=1}^n x_i D_i$, where