Remark 1.2.2: If $K = \mathbb{C}$, then we obtain from (1.15) and (1.16) that $Der(V) = so(K^{n+1}, b)$, the set of endomorphisms D of K^{n+1} with $b(Du, v) + b(u, Dv) = 0$. In fact, for any field K of characteristic 0 the Lie algebra of derivations of (K^{n+1}, b, f) is $so(K^{n+1}, b)$. This will be proved in the following.

Let K be an arbitrary field of characteristic 0. Let $V = (K^{n+1}, b, f)$. Let us describe $Der(V)$ and $Inder(V)$. By Proposition 1.2.2 and Theorem 1.1.3, $Der(V) = Inder(V)$. Hence we need to determine the inner derivations only.

We have seen in Example 1.1.1 that the elements of $so(K^{n+1}, b)$ are derivations of V (see (1.5)). Now let D be a derivation of V. For $v_1, \cdots, v_{n+1} \in V$ we get according to (1.4) and (1.9) for all $i \in \underline{n}$

$$
\begin{aligned}
b(D[v_1, \cdots, v_n], v_{n+1}) &= \sum_{i=1}^{n} b([v_1, \cdots, De_i, \cdots, v_n], v_{n+1}) \\
&= \operatorname{tr}(D)\, b([v_1, \cdots, v_n], v_{n+1}) - b([v_1, \cdots, v_n], D\, v_{n+1}).
\end{aligned}
$$

Since V is simple, $[V, \cdots, V] = V$, we get $b(Du, v) + b(u, Dv) = \operatorname{tr}(D)\, b(u, v)$. Since $\operatorname{tr}(D) = 0$ (see the proof of 3) at the beginn of this section), it follows that $b(Du, v) + b(u, Dv) = 0$, that is, D is an element in $so(K^{n+1}, b)$. Hence $Inder(V) = so(K^{n+1}, b)$.

For later use we summarize the main results in this section.

Theorem 1.2.4: *Let K be an algebraically closed field. Then there is only one simple n-Lie algebra of dimension $n + 1$ up to isomorphism. A realization of this n-Lie algebra is the vector product (K^{n+1}, b, f). If $\mathrm{char}K = 0$, then the Lie algebra of its derivations is $so(K^{n+1}, b)$ and all derivations are inner.*