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AT2) its left multiplications $z \to p(x, y, z)$ are derivations.

As we have seen above, Lie algebras and alternating triple systems are characterized by the properties that their products are alternating and their left multiplications are derivations. We can formally require these two properties for an n-linear product on a vector space. Then we have naturally generalized the concept of Lie algebras and alternating triple systems. Now, a vector space V with an n-linear map $p : \times^n V \to V$ is called an n-Lie algebra if

NL1) p is alternating and

NL2) all left multiplications $y \to p(x_1, \cdots, x_{n-1}, y)$ are derivations.

This definition of an n-Lie algebra was originally given by Filippov in [5]. In that paper there are also determined the $(n+1)$-dimensional n-Lie algebras and it is shown that every $(n+1)$-dimensional n-Lie algebras is isomorphic to one of the following n-Lie algebras on K^{n+1} given by the multiplication table:

$$[e_1, \cdots, e_{i-1}, e_{i+1}, \cdots, e_n] = \alpha_i e_i, \;\; i = 1, \cdots, n+1, \qquad (*)$$

where $\{e_1, \cdots, e_{n+1}\}$ is a base of K^{n+1} and $\alpha_i \in K$, $i = 1, \cdots, n+1$. Note that if K is algebraically closed, then all α_i can be chosen as 1 or 0.

Let us recall the definitions of solvability and semisimplicity given in [5]. We define recursively:

$$V^{(0)} := V, \quad V^{(s+1)} := [V^{(s)}, \cdots, V^{(s)}], s \in \mathbb{N}_0.$$

Then an n-Lie algebra is called solvable if $V^{(s)} = \{0\}$ for some s. An ideal of an n-Lie algebra is called solvable if it is solvable as an n-Lie algebra. As in case of Lie algebras a finite dimensional n-Lie algebra has a unique maximal solvable ideal called the radical of the given n-Lie algebra. If an n-Lie algebra possesses no nonzero solvable ideals, then it is called semisimple. In case $n = 2$ the above definitions agree with the usual definitions for Lie algebras. Therefore the concept of solvability and the concept of semisimplicity of n-Lie algebras generalize those of Lie algebras.

In this work we investigate the structure of finite dimensional n-Lie algebras over an algebraically closed field of characteristic 0. We are mainly interested in classifying n-Lie algebras. Now, at first we are concerned with the problem of finding the simple representatives of n-Lie algebras and of determining whether a semisimple n-Lie algebra is a direct sum of its simple ideals. These problems have a well-known beautiful answer in the case of Lie algebras. Here we will give a complete answer too. In fact, we shall prove that a finite dimensional n-Lie algebra