Schur's Lemma: *If K is algebraically closed and if V and W are irreducible L-modules for an arbitrary Lie algebra L, then the vector space of all L-module morphism from V to W is one dimensional if V and W are equivalent; and is $\{0\}$ if V an W are inequivalent.*

Let V be an arbitrary L-module. The dual space V^* of V becomes an L-module by $(x.f)v := -f(x.v)$ $(x \in L,\ f \in V^*,\ v \in V)$, to which we refer as the dual module V^* of V. If V is equivalent to its adjoint module V^*, then we say that V is self-contragredient. We can define an L-invariant bilinear form b on a self-contragredient L-module V as follows: $b(u, v) := (\tau u)(v)$, where τ is an L-module isomorphism from V onto V^*. If V is irreducible and self-contragredient, then there exists up to a scalar only one such form on V (cf. [14]). Moreover it is symmetric or skew-symmetric.

Given another Lie algebra L' and an L'-module W. Let $L_1 = L \oplus L'$. Then the tensor product $V \otimes W$ over K of the underlying vector spaces is an L_1-module with respect to $(x + y).(v \otimes w) := x.v \otimes w + v \otimes y.w$. This module is called the tensor product of L-modules V and and L'-module W. If $L' = L$, then we obtain an L-module by defining $x.(v \otimes w) := x.v \otimes w + v \otimes x.w$. This L-module is said to be the tensor product of L-modules V and W.

Let V be an L-module and $n \in \mathbb{N}$. Then the symmetric group S_n acts on $\otimes^n V$ via: $\sigma.(v_1 \otimes \cdots \otimes v_n) := v_{\sigma^{-1}(1)} \otimes \cdots \otimes v_{\sigma^{-1}(n)}$ for $\sigma \in S_n$, $v_i \in V$, $i \in \underline{n}$. The elements w of $\otimes^n V$ satisfying $\sigma.w = w$ (resp. $\sigma.w = sign(\sigma)\, w$) for all $\sigma \in S_n$ form a subspace $\vee^n V$ $(\wedge^n V)$ of $\otimes^n V$. One can prove that they are L-invariant subspaces of $\otimes^n V$.

Given a representation ρ of L in V, we call the symmetric bilinear form $\kappa : L \times L \to K$, defined by $\kappa(x, y) := \mathrm{tr}(\rho(x)\rho(y))$, the associated form of the representation (or the module) of L. If ρ is the adjoint representation, then the associated form is called the Killing form of L and denoted by $\mathrm{Kill}(\cdot, \cdot)$. In the following we give two theorems and some criteria for semisimplicity and reductivity.

Theorem A1: *(cf. [1] p. 50, p. 52 and p.53) Let L be a Lie algebra. Then the following conditions are equivalent:*

1) L is semisimple.

2) L is a direct sum of its simple ideals.

3) Each L-module is completely reducible.

4) The Killing form of L is nondegenerate.

Theorem A2: *(cf. [1] p. 56) The following are equivalent for a Lie algebra L:*