lowing we introduce direct sums of n-Lie algebras before we give a characterization of semisimple n-Lie algebras over a field of characteristic 0.

Let V_i be n-Lie algebras, $i = 1, 2$. It can easily be proved that the vector space direct sum $V = V_1 + V_2$ is an n-Lie algebra with regard to the product

$$[u_1 + v_1, \cdots, u_n + v_n] := [u_1, \cdots, u_n] + [v_1, \cdots, v_n] \quad u_i \in V_1,\ v_i \in V_2,\ i \in \underline{n}.$$

Evidently V_i, $i = 1, 2$, are ideals of V. We call the n-Lie algebra V the direct sum of the n-Lie algebras V_1 and V_2 and write $V = V_1 \oplus V_2$. This definition can be generalized to direct sum of m n-Lie algebras: $V = \oplus_{i=1}^{m} V_i$. In this situation we have

Theorem 2.5: *If $V = \oplus_{i=1}^{m} V_i$ is a direct sum of n-Lie algebras V_i, $i \in \underline{m}$, then*

1) $Inder(V) \cong \oplus_{i=1}^{m} Inder(V_i)$,

2) For all $k \in \underline{n}$: $Rad_k(V) = \oplus_{i=1}^{m} Rad_k(V_i)$.

For the following proposition we delete the assumption that the n-Lie algebra V is finite dimensional over the field K. For the sake of convenience we write L' for $Der(V)$ and L for $Inder(V)$. If M is a subspace of L' (or L) and I is a subspace of V, we denote by $M^s(I)$ the subspace of V which is spanned by all elements of the form $D_1 D_2 \cdots D_s(v)$, $D_i \in M$, $v \in I$.

Proposition 2.6: *Let V be an arbitrary n-Lie algebra. If M is an ideal of L' (or L) and I an ideal of V, then $M(I)$ is an ideal of V and $(M(I))^{(s,n)} \subseteq M^{s+1}(I)$ for $s \geq 1$.*

Proof: 1) $[M(I), V, \cdots, V] = L(M(I)) \subseteq [L, M](I) + M(L(I)) \subseteq M(I)$. Thus $M(I)$ is an ideal of V.

2) We prove the inclusion inductively. Since $M(I)$ and $M^2(I)$ are ideals of V by 1), it follows that

$$\begin{aligned}
&(M(I))^{(1,n)} = [M(I), \cdots, M(I)] \\
&\subseteq M([I, M(I), \cdots, M(I)]) + [I, MM(I), M(I), \cdots, M(I)] \\
&\subseteq M^2(I),
\end{aligned}$$

that is, $(*)$ $\quad (M(I))^{(1,n)} \subseteq M^2(I)$. Suppose the inclusion is true for s. Replacing I in $(*)$ by $M^s(I)$ we obtain: $(M^{s+1}(I))^{(1,n)} \subseteq M^{s+2}(I)$. By induction hypothesis, $(M(I))^{(s+1,n)} \subseteq ((M(I))^{(s,n)})^{(1,n)} \subseteq M^{s+2}(I)$. □

Now we are ready to give a characterization of finite dimensional semisimple n-Lie algebras over K of characteristic 0. We remark that a simple ideal of an n-Lie algebra is an ideal of the given n-Lie algebra which is simple as an n-Lie algebra.