By 2) this identity can be reformulated as

$$(\mu + \alpha\,\nu)[v_1, \cdots, v_n] = [(\mu + \alpha\,\nu)v_1, \cdots, (\mu + \alpha\,\nu)v_n],$$

this means that $\mu + \alpha\,\nu$ is an n-Lie algebra homomorphism from $\overline{V}$ to V. Since $\pi \circ (\mu + \alpha\,\nu) = id_{\overline{V}}$, the image of $\mu + \alpha\nu$ gives a Levi subalgebra of V. □

<u>**Theorem 4.2**</u>: *Let V be an n-Lie algebra over K. If V_0 is a Levi subalgebra of V, then V_0 is also a Levi subalgebra of $V^{(1,n)}$ ($= [V, \cdots, V]$) and $V^{(1,n)} = [V, \cdots, V, Rad(V)] + V_0$ is a Levi decomposition of $V^{(1,n)}$.*

Proof: Since $V = Rad(V) + V_0$ and $[V_0, \cdots, V_0] = V_0$, we have

$$\begin{aligned} V^{(1,n)} &= [Rad(V) + V_0, \cdots, Rad(V) + V_0] \\ &= [V, \cdots, V, Rad(V)] + [V_0, \cdots, V_0] \\ &= [V, \cdots, V, Rad(V)] + V_0 \end{aligned}$$

From $[V, \cdots, V, Rad(V)] \cap V_0 \subseteq Rad(V) \cap V_0 = \{0\}$, we obtain $Rad\ V^{(1,n)} = [V, \cdots, V, Rad(V)]$ and the assertion. □