($U(L)$ denotes the universal enveloping algebra of L), then we say briefly that V is standard cyclic and call λ the maximal weight of V.

Theorem A8: *Let V be a standard cyclic L-module with maximal vector $v^+ \in V_\lambda$. Let $\Phi^+ = \{\beta_1, \cdots, \beta_m\}$. Then:*

1) *V is spanned by the vectors $y_{\beta_1}^{i_1} \cdots y_{\beta_m}^{i_m} v^+$ ($i_j \in \mathbb{Z}^+$); in particular, V is the direct sum of its weight spaces.*

2) *The weights of V are of the form $\mu = \lambda - \sum_{i=1}^{l} k_i \alpha_i$ ($k_i \in \mathbb{Z}^+$), i.e., all weights of V satisfy $\mu \prec \lambda$.*

3) *For each weight μ of V, $\dim(V_\mu) < \infty$, and $\dim(V_\lambda) = 1$.*

4) *If V is an irreducible L-module, then v^+ is the unique maximal vector in V up to nonzero scalar multiples.*

For each $\lambda \in H^*$ there exists one and (up to isomorphism) only one irreducible standard cyclic L-module $V(\lambda)$ of maximal weight λ. $V(\lambda)$ is finite dimensional if and only if $\lambda \in \Lambda^+$. Let $\lambda \in \Lambda^+$ in the following. We write $\Pi(\lambda)$ for the set of weights of $V(\lambda)$. It holds that

$$\Pi(\lambda) = \{\sigma\mu \mid \sigma \in W,\ \mu \in \Lambda^+,\ \mu \prec \lambda\} = \{\mu \in \Lambda \mid \sigma\mu \prec \lambda, \forall \sigma \in W\},$$

and $\dim V_{\sigma\mu} = \dim V_\mu$ for all $\sigma \in W$ and all $\mu \in \Pi(\lambda)$. Let $\mu \in \Pi(\Lambda)$ and $\alpha \in \Phi$. If $r, q \in \mathbb{N}_0$ are maximal such that $\mu - r\alpha \in \Pi(\lambda)$ respectvely $\mu + q\alpha \in \Pi(\lambda)$, then

$$r - q = \langle \mu, \alpha \rangle. \tag{A.1}$$

Let $\sigma_0 \in W$ be as in Proposition 5. For any $\mu \in \Pi(\lambda)$, $\sigma_0\mu \in \Pi(\lambda)$. From $\lambda - \sigma_0\mu \succ 0$ it follows that $0 \succ \sigma_0(\lambda - \sigma_0\mu) = \sigma_0\lambda - \mu$. Hence $\mu - \sigma_0\lambda \succ 0$ and we call $\sigma_0\lambda$ the minimal weight of $V(\lambda)$. A weight vector $v^- \in V_{\sigma_0\lambda}$ which is not 0 is said to be minimal. A minimal vector v^- is characterized by the relation: $y_\alpha.v^- = 0$ for all $\alpha \succ 0$. Because of $\dim V_{\sigma_0\lambda} = 1$ any two minimal vectors are proportional.

By definition it is clear that the weights of the dual module $V^* := V(\lambda)^*$ (it is also irreducible) are $-\mu$, where $\mu \in \Pi(\lambda)$. In view of the above discussion $-\sigma_0\lambda$ must be the maximal weight of V^*. Therefore $V^* = V(-\sigma_0\lambda)$. So $V(\lambda)$ is self-contragredient if and only if $\sigma_0\lambda = -\lambda$.

Proposition A9: *Let L be a semisimple Lie algebra and α a positive root of L. Further let $V(\lambda)$ be an irreducible L-module with a maximal (minimal) vector v^+ (v^-). Then $y_\alpha.v^+ \neq 0$ if and only if $\langle \lambda, \alpha \rangle \neq 0$. Similarly $x_\alpha.v^- \neq 0$ if and only if $\langle \sigma_0\lambda, \alpha \rangle \neq 0$.*