$$\wedge^3 V(\lambda_3) \cong V(\lambda_1+\lambda_3) \oplus V(\lambda_3) \tag{3.9}$$
$$\wedge^4 V(\lambda_3) \cong V(\lambda_1) \oplus V(2\lambda_1) \oplus V(2\lambda_3) \oplus V(0) \tag{3.10}$$
$$\wedge^5 V(\lambda_3) \cong V(\lambda_1+\lambda_3) \oplus V(\lambda_3) \tag{3.11}$$
$$\wedge^6 V(\lambda_3) \cong V(\lambda_1) \oplus V(\lambda_2) \tag{3.12}$$
$$\wedge^7 V(\lambda_3) \cong V(\lambda_3) \tag{3.13}$$
$$\vee^2 V(\lambda_3) \cong V(2\lambda_3) \oplus V(0) \tag{3.14}$$

Notice that $V(\lambda_2)$ is the adjoint module of $so(7,K)$. According to these decompositions we could have at most two good triples from $so(7,K)$ and $V(\lambda_3)$. (see (3.8) and (3.12)).

At first we check wether $(so(7,K), V(\lambda_3), \tau)$, where $\tau$ is an $so(7,K)$-module morphism from $\wedge^2 V(\lambda_3)$ onto $so(7,K)$ ($\tau$ is unique up to scalar), is a good triple. Since there is an $L$-invariant nondegenerate symmetric bilinear form on $V(\lambda_3)$ (see (3.14)), we can show that there is no 3-Lie algebra on $V(\lambda_3)$ such that the Lie algebra of its derivations is isomorphic to $so(7,K)$ as in Case 1.

We now study whether there exists an 7-Lie algebra on $V(\lambda_3)$ with its derivation algebra isomorphic to $so(7,K)$. Theorem 1.2.4 shows each 7-Lie algebra on $V(\lambda_3)$ is isomorphic to the vector product $(K^8, b, f)$. But the derivation algebra of the vector product is $so(8,K)$. Hence there is no good triple of the form $(so(7,K), V(\lambda_3), \tau)$.

*Case 5:* $L \cong D_4$, $\lambda = \lambda_3$ *or* $\lambda_4$.

$V(\lambda_3)$ and $V(\lambda_4)$ are the two 8-dimensional spin modules of $so(8,K)$. Since there is an automorphism $\tau$ of $so(8,K)$ such that the natural module $K^8$ becomes $V(\lambda_3)$ or $V(\lambda_3)$ via $x.v := (\tau x)(v)$ or $x.v := (\tau^2 x)(v)$ for $x \in so(8,K)$ and $v \in K^8$. In view of the equivalence of good triples we are led to Case 1.

*Case 6:* $L \cong G_2$, $\lambda = \lambda_1$.

$V(\lambda_1)$ is the natural 7-dimensional $G_2$-module. $V(\lambda_2)$ is the adjoint module.

$$\wedge^2 V(\lambda_1) \cong V(\lambda_1) \oplus V(\lambda_2) \tag{3.15}$$
$$\wedge^3 V(\lambda_1) \cong V(2\lambda_1) \oplus V(\lambda_1) \oplus V(0) \tag{3.16}$$
$$\wedge^4 V(\lambda_1) \cong V(2\lambda_1) \oplus V(\lambda_1) \oplus V(0) \tag{3.17}$$
$$\wedge^5 V(\lambda_1) \cong V(\lambda_1) \oplus V(\lambda_2) \tag{3.18}$$
$$\wedge^6 V(\lambda_1) \cong V(\lambda_1) \tag{3.19}$$
$$\vee^2 V(\lambda_1) \cong V(2\lambda_1) \oplus V(0) \tag{3.20}$$

Since $V(\lambda_1)$ is a self-contragredient $G_2$-module and there is up to scalar only one nonzero $G_2$-module morphism from $\wedge^2 V(\lambda_1)$ onto $G_2$ (see (3.15) and (3.20)), we can show as in Case 1 that there is no 3-Lie algebra on $V(\lambda_1)$ with its derivation