for all $i \in \underline{m}$. If $m = 1$, then we done because $V_0 = V_1$. Now let $m \geq 2$. We will show that $V_{(n_1^0,\cdots,n_m^0)} = \{0\}$ which implies that V_0 is a subalgebra of V and therefore a Levi subalgebra.

By renumbering we can assume that $0 < n_1^0 < n-1$. Then because of 9) we have for all $2 \leq i \leq m$,

$$[\underbrace{V_1, \cdots, V_1}_{n-1}, V_i] = \{0\}, \tag{4.1}$$

since $(n_1^0, \cdots, n_m^0) \neq (n-1, 0, \cdots, 1, \cdots, 0)$. Since $\pi_1(V_1) = \overline{V}_1$, V_1 is $(n+1)$-dimensional simple n-Lie algebra. Let $\{e_1, \cdots, e_{n+1}\}$ be a basis of V_1 with

$$[e_1, \cdots, \widehat{e_i}, \cdots, e_{n+1}] = e_i, \quad i \in \underline{n+1} \tag{4.2}$$

(see [5]). Let $v_{i,j} \in V_i$, $2 \leq i \leq m$, $j \in \underline{n_i^0}$. Let v_j, $j \in \underline{n_1^0}$ be arbitrary distinct elements in $\{e_1, \cdots, e_{n+1}\}$. By an appropriate numbering we can assume that $v_j = e_j$, $j \in \underline{n_1^0}$. Then we have by (4.1), (4.2) and the G.J.I.,

$$\begin{aligned}
&[e_1, \cdots, e_{n_1^0}, v_{2,1}, \cdots, v_{2,n_2^0}, \cdots, v_{m,1}, \cdots, v_{m,n_m^0}] \\
&= [e_1, \cdots, e_{n_1^0-1}, [e_1, \cdots, \widehat{e_{n_1^0}}, \cdots, e_{n+1}], v_{2,1}, \cdots, v_{m,n_m^0}] \\
&= \sum_{s=n_1^0+1}^{n+1} [e_1, \cdots, \widehat{e_{n_1^0}}, \cdots, [e_1, \cdots, e_{n_1^0-1}, e_s, v_{2,1}, \cdots, v_{m,n_m^0}], \cdots, e_{n+1}] \\
&= \sum_{s=n_1^0+1}^{n+1} [e_1, \cdots, e_{n_1^0-1}, [e_1, \cdots, \widehat{e_{n_1^0}}, \cdots, e_{n+1}], v_{2,1}, \cdots, v_{m,n_m^0}] \\
&= \sum_{s=n_1^0+1}^{n+1} [e_1, \cdots, e_{n_1^0}, v_{2,1}, \cdots, v_{m,n_m^0}] \\
&= (n+1-n_1^0)[e_1, \cdots, e_{n_1^0}, v_{2,1}, \cdots, v_{m,n_m^0}].
\end{aligned}$$

Because $n_1^0 < n$, we get $[e_1, \cdots, e_{n_1^0}, v_{2,1}, \cdots, v_{m,n_m}] = 0$, which implies the relation $V_{(n_1,\cdots,n_m)} = \{0\}$.

Case 2.3: I is equivalent to some V_i, say V_1.

By the assumption, I is equivalent to $\overline{V}_1$. Notice that $\overline{V}_1$ is the only $\overline{L}$-submodule of $\overline{V}$ that is equivalent to I by the assumption. Therefore all nonzero $\overline{L}$-module morphisms from $\overline{V}$ to I are proportional to each other by Schur's Lemma. The same is true for $\overline{L}$-module morphisms from $\wedge^n \overline{V}$ to I in view of the following