Let b_0 be the restriction of b to V_0 and f_0 be the nonzero determinant form on V_0 defined by $f_0(v_1, \cdots, v_n) := -f(v_1, \cdots, v_n, x)$. Then b_0 is a nondegenerate symmetric bilinear form on V_0 and for all $v_i \in V_0$, $i \in \underline{n}$ we have

$$\begin{aligned}
b_0([v_1, \cdots, v_{n-1}]_x, v_n) &= b_0([v_1, \cdots, v_{n-1}, x], v_n) \\
&= f(v_1, \cdots, v_{n-1}, x, v_n) \\
&= -f(v_1, \cdots, v_{n-1}, v_n, x) \\
&= f_0(v_1, \cdots, v_{n-1}, v_n).
\end{aligned}$$

This means that the $(n-1)$-Lie algebra V_0 can be realized as (K^n, b_0, f_0). Therefore V_0 is simple by Proposition 1.2.2. It follows that Kx is the centre of $V(x)$ and $V(x)$ is reductive.

Now let K be algebraically closed. Let $y \in V$, $y \neq x$, be such that $b(y,y) \neq 0$. We may ask whether the reductive $(n-1)$-Lie algebras $V(x)$ and $V(y)$ are isomorphic. Let $V(y) = Ky \oplus V_1$. Since V_0 and V_1 are simple $(n-1)$-Lie algebras, there exists an $(n-1)$-Lie algebra isomorphism T from V_0 onto V_1 (see Theorem 1.2.4). Extend T to $V(x)$ linearly via:

$$T_{x,y}(v + \alpha x) := T(v) + \alpha y, \quad v \in V_0,\ \alpha \in K.$$

One can easily check that $T_{x,y}$ is an $(n-1)$-Lie algebra isomorphism.

We can also construct an isomorphism from $V(x)$ onto $V(y)$ as follows. Since $b(x,x) \neq 0$ and $b(y,y) \neq 0$, there exists a scalar $\alpha \in K$, $\alpha \neq 0$ and an element $\tau \in O(K^{n+1}, b)$ such that $y = \alpha\,\tau(x)$. Let $\tau' := (\alpha \det(\tau))^{-\frac{1}{n-2}}\tau$. From

$$\begin{aligned}
b([\tau' v_1, \cdots, \tau' v_{n-1}]_y, v_n) &= (\alpha \det(\tau))^{-\frac{n-1}{n-2}} b([\tau v_1, \cdots, \tau v_{n-1}]_y, v_n) \\
&= (\alpha \det(\tau))^{-\frac{n-1}{n-2}} f(\tau v_1, \cdots, \tau v_{n-1}, y, v_n) \\
&= (\alpha \det(\tau))^{-\frac{1}{n-2}} f(v_1, \cdots, v_{n-1}, x, \tau^{-1} v_n) \\
&= (\alpha \det(\tau))^{-\frac{1}{n-2}} b([v_1, \cdots, v_{n-1}]_x, \tau^{-1} v_n) \\
&= (\alpha \det(\tau))^{-\frac{1}{n-2}} b(\tau[v_1, \cdots, v_{n-1}]_x, v_n) \\
&= b(\tau'[v_1, \cdots, v_{n-1}]_x, v_n)
\end{aligned}$$

it follows that $\tau'[v_1, \cdots, v_{n-1}]_x = [\tau' v_1, \cdots, \tau' v_{n-1}]$ for all $v_i \in V$, $i \in \underline{n-1}$, that is, τ' is an isomorphism from $V(x)$ onto $V(y)$.

In the following theorem we give two criteria for reductivity.

Theorem 2.10: *Let K be of characteristic 0 and V be an n-Lie algebra V over K. The following are equivalent.*

1) $Inder(V)$ is semisimple.