Introduction

In this work we will study a generalization of the concept of Lie algebras. In order to give the reader an idea how this generalization arises we are going to motivate our study by first recalling the concept of Lie algebras. A Lie algebra is by definition a vector space L with a bilinear product $[x,y]$, such that for all $x,y,z \in L$

L1) $[y,x] = -[x,y]$,

L2) $[x,[y,z]] = [[x,y],z] + [y,[x,z]]$ (Jacobi's identity).

Clearly the Jacobi identity is equivalent to the condition that the left multiplication $\mathrm{ad}x : L \to L$ defined by $\mathrm{ad}x(y) = [x,y]$ is a derivation of L. Therefore we can reformulate the definition of Lie algebras as follows: An algebra L is a Lie algebra if

L1') its product is alternating, and

L2') every left multiplication is a derivation.

We also recall the concept of triple systems. A triple system is simply a vector space T with a trilinear product $p : T \times T \times T \to T$. In 1985 Faulkner [3] discussed a new class of triple systems which he called alternating triple systems. He was led to this type of triple systems by studying which trilinear identities are satisfied by a nearly simple triple system over an algebraically closed field K of characteristic 0, i.e. by a triple system over K such that

1) its left multiplications are derivations,

2) its derivation algebra acts irreducibly on it.

To be more concrete, let T be an arbitrary triple system with product p. An element $u = \sum \alpha_\sigma \sigma$ in the group algebra $K[S_3]$, is viewed as a trilinear identity if $u(p)(x_1,x_2,x_3) = \sum \alpha_\sigma p(x_{\sigma 1}, x_{\sigma 2}, x_{\sigma_3}) = 0$. Clearly the trilinear identities satisfied by p form a left ideal I_p in $K[S_3]$. It is proved in [3] that if T is a nontrivial, nearly simple triple system, then I_p contains at least one of the left ideals corresponding to six types of triple systems which are called: left-skew, left-symmetric, Anti-Lie, Lie, Jacobi and alternating triple system (see [3,Thm 9]). This reduces the study of the infinite family of inequivalent identities to the six types above.

Furthermore Faulkner has proved that the alternating triple system on the space of quaternions with triple product $x\overline{y}z - z\overline{y}x$ is the only nontrivial, alternating, nearly simple triple system up to isomorphism.

Recall that an alternating triple system is by definition a triple sytem such that