Proof of Theorem 4.1:

We will set $I := Rad(V)$ in the following. If $I = \{0\}$, then V is semisimple and we are done. So let $I \neq \{0\}$ and suppose that the assertion is true in case I is a minimal ideal of V. Under this hypothesis we prove the existence of a Levi subalgebra for any n-Lie algebra by induction on the dimension of I. Assume that the assertion is true for all n-Lie algebras whose radical has dimension less than $\dim I$. If I is a minimal ideal of V, then we are done. Now assume that I is not minimal. Let $J \neq \{0\}$ be an ideal of V which is properly included in I and $\pi : V \to V/J$ be the canonical map. Then $\pi(I)$ is a solvable ideal of V/J by Proposition 2.2, 2). Since $\pi^{-1}(Rad(V/J))$ is a solvable ideal of V by Proposition 2.2 3), it is included in I. Hence $Rad(V/J) \subseteq \pi(I)$ and $\pi(I)$ coincides with the radical of V/J. Since $\dim\pi(I) < \dim I$, there exists a Levi subalgebra V' of V/J with $V/J = \pi(I) + V'$ and $\pi(I) \cap V' = \{0\}$ by the induction hypothesis. Set $V_1 := \pi^{-1}(V')$. Then V_1 is a subalgebra of V with $V = I + V_1$. Moreover, we have $I \cap V_1 = J$. In fact, from $\pi(I \cap V_1) \subseteq \pi(I) \cap \pi(V_1) = \pi(I) \cap V' = \{0\}$, we obtain that $I \cap V_1 \subseteq J$. The other inclusion is evident. We show that J is the radical of V_1. Since J is a solvable ideal of V, it is a solvable ideal of V_1, and thus $J \subseteq Rad(V_1)$. On the other hand, it follows from $\pi(Rad(V_1)) = \{0\}$ that $Rad(V_1) \subseteq J$, hence $J = Rad(V_1)$. Since $\dim J < \dim I$, we can find a Levi subalgebra V_0 of V_1 by the induction hypothesis. Then V_0 is also a Levi subalgebra of V because $V = I + (J + V_0) = I + V_0$ and $I \cap V_0 = \{0\}$.

It remains to show Theorem 4.1 in case I is a minimal ideal of V.

We set $\overline{V} := V/I$ and let $\pi : V \to \overline{V}$ be the canonical homomorphism. Let L (resp. $\overline{L}$) be the derivation algebra of V (resp. $\overline{V}$). If $\overline{V} = \{0\}$, then V is solvable and we are done. Assume that $\overline{V} \neq \{0\}$. Since $\overline{V}$ is a semisimple n-Lie algebra, it is the direct sum of its simple ideals, say $\overline{V} = \oplus_{i=1}^{m} \overline{V}_i$ for some $m \in \mathbb{N}$ (see Theorem 2.7). Moreover $\overline{L} \cong \oplus_{i=1}^{m} \overline{L}_i$, $\overline{L}_i = Inder(\overline{V}_i) \cong so(n+1, K)$ (see Theorem 1.2.4, Theorem 2.5 and Theorem 3.9). Let $\gamma : L \to \overline{L}$ be the Lie algebra homomorphism defined as in Theorem 2.9. Recall that γ is surjective with $Ker\gamma = \{D \in L \mid D(V) \subseteq I\}$ and $R \subseteq Ker\gamma$, where R denotes the radical of L. Because for any Lie algebra L and any ideal M of L we have $Rad(M) = Rad(L) \cap M$ (cf. [18] p. 204), R is the radical of $Ker\gamma$. Let L_1 be a Levi subalgebra of $Ker\gamma$. Since L_1 is semisimple, there exists a Levi subalgebra L_0 of L such that $L_1 \subseteq L_0$ (cf. [18] p. 226 and 228). In fact, L_1 is an ideal of L_0. To see this we first show that $L_0 \cap Ker\gamma = L_1$. Let $x \in L_0 \cap Ker\gamma$ and $x = y + z$, $y \in R$, $z \in L_1$. Because of $yx - z$ and $x - z \in L_0$ we must have $y \in L_0$, hence $y \in R \cap L_0 = \{0\}$ which implies that $x \in L_1$, that is $L_0 \cap Ker\gamma \subseteq L_1$. The other inclusion is trivial. Now, since L_0 is a subalgebra and $Ker\gamma$ is an ideal of L, it follows that $[L_0, L_1] \subseteq L_0 \cap Ker\gamma = L_1$,