1) L is reductive.

2) $L = C \oplus [L, L]$, where C is the centre of L and $[L, L]$ is semisimple.

3) There exists a faithful completely reducible L-module.

4) There exists a representation ρ of L whose associated form is nondegenerate.

5) L is completely reducible as an L-module.

Let P be an algebraic closure of K. Then $P \otimes_K L$, regarded as a vector space over P, is a Lie algebra relative to the product

$$[\alpha \otimes x, \beta \otimes y] := \alpha\beta \otimes [x, y].$$

If $K = \mathbb{R}$ and $P = \mathbb{C}$, then we refer to the Lie algebra $\mathbb{C} \otimes_{\mathbb{R}} L$ as the complexification of L.

__Theorem A3__: *([13] p. 261) L is semisimple over K if and only if $P \otimes_K L$ is semisimple over P.*

If L is a subalgebra of $gl(V)$, where V is a vector space over K, then $P \otimes_K L$ can be regarded as a subalgebra of $gl(P \otimes_K V)$.

__Thereom A4__: *([13] p. 251) L is completely reducible in V over K if and only if $P \otimes_K V$ is completely reducible in $P \otimes_K V$ over P.*

With the help of the last two theorems many problems can be reduced to the case that the ground field K is algebraically closed. In the following we assume that K is such a field.

A very powerful means to describe semisimple Lie algebras is the theory of root systems. Let E be a Euclidean space with scalarproduct $(\cdot, \cdot) : E \times E \longrightarrow R$. For an $\alpha \in E$, $\alpha \neq 0$ we define a linear map $\sigma_\alpha : E \longrightarrow E$ via

$$\sigma_\alpha(\beta) := \beta - \frac{2(\beta, \alpha)}{(\alpha, \alpha)}\alpha \qquad \forall \beta \in E.$$

For brevity we set $\langle \beta, \alpha \rangle := \frac{2(\beta,\alpha)}{(\alpha,\alpha)}$. The map σ_α clearly satisfies the following properties: $\sigma_\alpha(\alpha) = -\alpha$, $\sigma_\alpha(\beta) = \beta$ for all $\beta \in P_\alpha := \{\gamma \in E \mid (\gamma, \alpha) = 0\}$ and $(\sigma_\alpha(\beta), \sigma_\alpha(\gamma)) = (\beta, \gamma)$ for all $\beta, \gamma \in E$. We call σ_α the reflection with respect to the hyperplane P_α. A root system in E is a set Φ with the following properties:

1) $|\Phi| < \infty$, Φ spans E and $0 \notin \Phi$.

2) If $\alpha \in E$, then $k\alpha \in \Phi$ implies $k = \pm 1$.