Theorem A6: *Let L be a semisimple Lie algebra and $L = \oplus_{i=1}^{s} L_i$, where L_i are ideals of L, $i \in \underline{s}$.*

1) *If H is a maximal toral subalgebra of L, then $H_i := H \cap L_i$ is a maximal toral subalgebra of L_i. If Φ is the root system of L relative to H and $\Phi^{(i)}$ is the set of the elements $\alpha \in \Phi$ with $\alpha(H_i) \neq \{0\}$, then*

$$\Phi^{(i)}|_{H_i} := \{\alpha|_{H_i} \mid \alpha \in \Phi^{(1)}\}$$

is the root system of L_i relative to H_i. Moreover $\Phi = \cup_{i=1}^{s} \Phi^{(i)}$ and $H = \oplus H_i$.

2) *If H_i is a maximal toral subalgebra of L_i and Φ_i the root system of L_i relative to H_i, then $H := \oplus H_i$ is a maximal toral subalgebra of L and $\Phi := \cup_{i=1}^{s} \Phi^{(i)}$ is the root system of L relative to H, where*

$$\Phi^{(i)} := \{\alpha \in H^* \mid \alpha|_{H_i} \in \Phi_i, \alpha|_{H_j} = 0, \forall j \neq i\}.$$

Remark A1: In view of Theorem A6 1) we shall identify the root system Φ_i of L_i relative to H_i with the subset $\Phi^{(i)}$ of Φ for a given maximal toral subalgebra H of L. Further we shall take the set $\Delta_i := \Phi^{(i)} \cap \Delta$ as a base of Φ_i.

Theorem A7: *A root system Φ of a simple Lie algebra is irreducible, that is, if $\Phi = \Phi_1 \cup \Phi_2$ with $(\Phi_1, \Phi_2) = \{0\}$, then $\Phi_1 = \Phi$ or $\Phi_2 = \Phi$.*

At most two lengths of roots occur in a root system of a simple Lie algebra. Among the roots of the same length there is a highest element relative to "$\prec$". In Table 1 we give the simple Lie algebras by their Dynkin diagram; in Table 2 the highest long and short roots (the numbering of the simple roots is as in Humphreys [7]). They are the only roots which are dominant. The highest long root will also be called the maximal root of L.

In the following let L be a semisimple Lie algebra over K, H a maximal toral subalgebra of L, Φ the root system of L relative to H, Δ a base of Φ, W the Weyl group of Φ. For later use we choose for each $\alpha \in \Phi$ three elements $x_\alpha \in L_\alpha$, $y_\alpha \in L_{-\alpha}$ and $h_\alpha \in H$ such that

$$[x_\alpha, y_\alpha] = h_\alpha,\ [h_\alpha, x_\alpha] = 2x_\alpha,\ [h_\alpha, y_\alpha] = -2y_\alpha.$$

For this choice we have $h_\alpha = \frac{2t_\alpha}{\mathrm{Kill}(t_\alpha, t_\alpha)}$. Therefore $\lambda(h_\alpha) = \langle \lambda, \alpha \rangle$ for any $\lambda \in \Lambda$.

Now suppose V is a (not necessarily finite dimensional) L-module. For a $\mu \in H^*$ let $V_\mu := \{v \in V \mid h.v = \mu(h)v,\ \forall h \in H\}$. If $V_\mu \neq \{0\}$, then V_μ is called a weight space of V and μ a weight of V. A maximal vector in V with respect to H is a nonzero vector $v^+ \in V_\lambda$ (of weight λ) killed by all x_α ($\alpha \in \Phi^+$). If $V = U(L).v^+$