

2) The vector space  $V/I$  is an  $n$ -Lie algebra relative to the product

$$[\pi v_1, \dots, \pi v_n] := \pi[v_1, \dots, v_n],$$

where  $\pi : V \rightarrow V/I$  denotes the canonical map.

3) Let  $\psi : V \rightarrow V'$  be an  $n$ -Lie algebra homomorphism.

(a) If  $I'$  is an ideal of  $V'$ , then  $\psi^{-1}I'$  is an ideal of  $V$ , in particular,  $\text{Ker}\psi$  is an ideal of  $V$ . Moreover  $\psi$  induces an isomorphism  $\tilde{\psi} : V/\text{Ker}\psi \rightarrow \text{Im}\psi$ . If  $I$  is any ideal of  $V$  included in  $\text{Ker}\psi$ , then there exists a unique homomorphism  $\phi : V/I \rightarrow V'$  such that  $\psi = \phi\pi$ .

(b) If  $\psi$  is in addition surjective, then the image of any ideal of  $V$  under  $\psi$  is an ideal of  $V'$ .

4) If  $J \subseteq I$ , then  $I/J$  is an ideal of  $V/J$  and  $(V/J)/(I/J)$  is naturally isomorphic to  $V/I$ .

5)  $(I + J)/J \cong I/(I \cap J)$ .

**Definition:** Let  $V$  be an  $n$ -Lie algebra over  $K$ . We call an endomorphism  $D$  of  $V$  a derivation if for all  $v_i \in V$ ,  $i \in \underline{n}$ :

$$D[v_1, \dots, v_n] = \sum_{i=1}^n [v_1, \dots, v_{i-1}, Dv_i, v_{i+1}, \dots, v_n]. \quad (1.9)$$

In consequence of identity (1.2) each left multiplication is a derivation. We refer to an endomorphism  $D$  of  $V$  as an inner derivation if it may be written as a sum of some left multiplications. We denote by  $\text{Der}(V)$  ( $\text{Inder}(V)$ ) the set of all (inner) derivations of  $V$ . One can verify

**Proposition 1.1.2:** Relative to the Lie bracket  $\text{Der}(V)$  is a Lie algebra and  $\text{Inder}(V)$  is an ideal of  $\text{Der}(V)$ .

Let  $V$  be an arbitrary  $n$ -Lie algebra and let  $U_i$ ,  $i \in \underline{n-1}$ , be subspaces of  $V$ . We denote by  $\text{ad}(U_1, \dots, U_{n-1})$  the subspace of  $\text{Inder}(V)$  spanned by the left multiplications  $\text{ad}(u_1, \dots, u_{n-1})$ ,  $u_i \in U_i$ . If  $U_i$ ,  $i \in \underline{n-1}$ , are ideals of  $V$ , then  $\text{ad}(U_1, \dots, U_{n-1})$  is an ideal of  $\text{Inder}(V)$ .

Let  $V$  be an arbitrary  $n$ -Lie algebra. Then the Lie algebras  $\text{Der}(V)$  and  $\text{Inder}(V)$  operate in a natural way on  $V$ , so we have a representation of them on  $V$ , or equivalently,  $V$  is a  $\text{Der}(V)$ - and an  $\text{Inder}(V)$ -module. If  $I$  is an ideal of  $V$ , that is,  $[I, V, \dots, V] \subseteq I$ , then  $I$  is an  $\text{Inder}(V)$ -submodule of  $V$ . Conversely if