$$= \alpha_i\beta_{j_0}\, e_i + (-1)^{j_0-i-1}\alpha_{j_0}\beta_i\, e_{j_0}.$$

Let $i_0 \le k$ such that $i_0 \ne i$, $i_0 \ne j_0$. Then

$$[e_1, \cdots, \widehat{e_{i_0}}, \cdots, e_{i-1}, \alpha_i\beta_{j_0}\, e_i + (-1)^{j_0-i-1}\alpha_{j_0}\beta_i\, e_{j_0}, e_{i+1}, \cdots, e_{n+1}] = \alpha_{i_0}\alpha_i\beta_{j_0}e_{i_0}.$$

Therefore $e_{i_0} \in J$. This forces $e_i \in J$ for all $i \le k$. Hence $I \subseteq J$.

Since I is not $(k-1)$-solvable ideal of V, V possesses no nonzero $(k-1)$-solvable ideal of V if $k \ge 3$.

In the following proposition we give some properties of k-solvable ideals.

Proposition 2.2: *Let V be an n-Lie algebra and $k \in \underline{n}$.*

1) *If V is k-solvable , then all subalgebras of V are k-solvable and all ideals of V are k-solvable ideals of V.*

2) *Let $\psi : V \longrightarrow V_1$ be a surjective n-Lie algebra homomorphism and I a k-solvable ideal of V, then ψI is a k-solvable ideal of V_1.*

3) *Let I, J be ideals of V, $J \subseteq I$. If J is a k-solvable ideal of V and I/J a k-solvable ideal of V/J, then I is a k-solvable ideal of V. In particular, if J is a k-solvable ideal of V such that V/J is a k-solvable n-Lie algebra, then V itself is k-solvable.*

4) *If I and J are k-solvable ideals of V, so is $I + J$.*

Proof: 1) If U is a subalgebra of V, then $U^{(s,k)} \subseteq V^{(s,k)}$. This implies the first assertion. If I is an ideal of V, then $I^{(s,k)} \subseteq V^{(s,k)}$. From it the second assertion follows.

2) It can be shown inductively that $\psi(I)^{(s,k)} = \psi(I^{(s,k)})$. This implies the assertion.

3) Denote the canonical homomorphism $V \longrightarrow V/J$ by π. Since I/J is a k-solvable ideal of V/J, we have $\pi(I^{(s,k)}) = \pi(I)^{(s,k)} = (I/J)^{(s,k)} = \{0\}$ for some s, hence $I^{(s,k)} \subseteq J$. But $J^{(s',k)} = \{0\}$ for some $s' \in \mathbb{N}$, so $I^{(s+s',k)} = (I^{(s,k)})^{(s',k)} \subseteq J^{(s',k)} = \{0\}$. Therefore I is a k-solvable ideal of V.

4) We consider the canonical homomorphism $\pi : V \longrightarrow V/J$. Since I is a k-solvable ideal of V, so $I/J = \pi(I)$ is a k-solvable ideal of V/J in view of 2). But $\pi^{-1}(I/J) = I + J$, so $I + J$ is a k-solvable ideal of V in view of 3). □

From now on we assume that V is finite dimensional over K. Let I be a maximal k-solvable ideal of V and J an arbitrary k-solvable ideal of V. According to Proposition 2.2 4) $I + J$ is also a k-solvable ideal of V which in turn implies