Hence each summand in the first sum cancels with the corresponding one in the second sum and therefore $\sum_{i=1}^{n} Dx_i = 0$. □

As a concrete example take A to be the real algebra $C^\infty(\mathbb{R}^n)$ of C^∞-functions on $\mathbb{R}^n$ and $D_i = \frac{\partial}{\partial x_i}$, $i \in \underline{n}$, where $n \geq 2$. Then we get an n-Lie algebra on $C^\infty(\mathbb{R}^n)$ with the product $[g_1, \cdots, g_n] := \left|\frac{\partial g_j}{\partial x_i}\right|$.

Example 1.1 and 1.2 can also be found in Filippov [5].

<u>**Example 1.1.3**</u>: Let V be an n-dimensional vector space over K, f a nonzero determinant form on V and $0 \neq v_0 \in V$. Then V becomes an n-Lie algebra relative to the product $[v_1, \cdots, v_n] := f(v_1, \cdots, v_n)\, v_0$.

In fact, by using Cramer's rule

$$f(v_1, \cdots, v_n)v_0 = \sum_{i=1}^{n} f(v_1, \cdots, v_{i-1}, v_0, v_{i+1}, \cdots, v_n)v_i,$$

we get

$$\begin{aligned}
&\sum_{i=1}^{n}[v_1, \cdots, v_{i-1}, [u_1, \cdots, u_{n-1}, v_i], v_{i+1}, \cdots, v_n] \\
&= \sum_{i=1}^{n} f(u_1, \cdots, u_{n-1}, v_i)[v_1, \cdots, v_{i-1}, v_0, v_{i+1}, \cdots, v_n] \\
&= \sum_{i=1}^{n} f(u_1, \cdots, u_{n-1}, v_i) f(v_1, \cdots, v_{i-1}, v_0, v_{i+1} \cdots, v_n)v_0 \\
&= f(u_1, \cdots, u_{n-1}, \sum_{i=1}^{n} f(v_1, \cdots, v_{i-1}, v_0, v_{i+1} \cdots, v_n)v_i)v_0 \\
&= f(u_1, \cdots, u_{n-1}, f(v_1, \cdots, v_n)v_0)v_0 \\
&= f(u_1, \cdots, u_{n-1}, [v_1, \cdots, v_n])v_0 \\
&= [u_1, \cdots, u_{n-1}, [v_1, \cdots, v_n]],
\end{aligned}$$

which gives the G.J.I..

<u>**Remark 1.1.1**</u>: Let V be an n-Lie algebra ($n > 2$) with product $[v_1, \cdots, v_n]$. For any fixed $v_0 \in V$ we may define an $(n-1)$-Lie algebra structure on the underlying vector space V by $[v_1, \cdots, v_{n-1}] := [v_1, \cdots, v_{n-1}, v_0]$. Although very easy to prove, it demonstrates us how to construct an $(n-1)$-Lie algebra from a given n-Lie algebra.

<u>**Definition**</u>: *Let V_1, V_2 be n-Lie algebras over K. A linear map $\tau : V_1 \to V_2$ is called*