algebra isomorphic to G_2. Therefore such an n-Lie algebra is necessarily a 6-Lie algebra (see (3.18)). But the derivation algebra of a simple 6-Lie algebra on a 7-dimensional vector space is isomorphic to $so(7, K)$ and therefore is 21-dimensional. Now $\dim(G_2) = 14 < 21 = \dim(so(7, K))$. Therefore we obtain no good triple in this case.

Summarizing the results from Case 1 to Case 6 we get

Theorem 3.8: *If V is a simple finite dimensional n-Lie algebra over an algebraically closed field of characteristic 0 such that the Lie algebra of its derivations is simple, then $n \geq 4$. Moreover V is isomorphic to the vector product (K^{n+1}, b, f).*

Combining Theorem 3.6 and 3.8, we obtain the following

Theorem 3.9: *For every $n \geq 3$ all finite dimensional simple n-Lie algebras over an algebraically closed field K of characteristic 0 are isomorphic to the vector product on K^{n+1}.*

As an application of Theorem 3.9 we will classify the finite dimensional real simple n-Lie algebras. The idea is analogous as in the case of Lie algebras, i.e. we investigte the complexified n-Lie algebras.

Let V be an arbitrary real n-Lie algebra. We form the tensor product $\widetilde{V} := \mathbb{C} \otimes_{\mathbb{R}} V$ and regard it as a vector space over $\mathbb{C}$: $z(z' \otimes v) := zz' \otimes v$. Obviously $\widetilde{V}$ is an n-Lie algebra with

$$[z_1 \otimes v_1, \cdots, z_n \otimes v_n] = z_1 \cdots z_n \otimes [v_1, \cdots, v_n].$$

This complex n-Lie algebra is called the complexification of V. We can formally think of $\widetilde{V}$ as

$$\widetilde{V} = \{u + iv \mid u, v \in V \text{ and } i^2 = -1\}.$$

Note that $V \subseteq \widetilde{V}$ by identifying V with $V + i\{0\}$. Let C be the map from $\widetilde{V}$ into itself with $C(u + iv) = u - iv$. Then V is just the set of fixed points of C.

Conversely, given a complex n-Lie algebra $\widetilde{V}$, then by restricting the ground field to the real numbers we obtain a real n-Lie algebra $\widetilde{V}_{\mathbb{R}}$, which will be called the realification of $\widetilde{V}$.

A real n-Lie algebra V is called a real form of $\widetilde{V}$ if its complexification is isomorphic to $\widetilde{V}$.

Proposition 3.10: *Let $\widetilde{V}$ be an arbitrary complex simple n-Lie algebra. Then the realification $\widetilde{V}_{\mathbb{R}}$ of $\widetilde{V}$ is simple.*