Now, since the root $\lambda - \sigma_0\lambda$ of the simple Lie algebra L is a sum of two nonzero dominant weights and since the roots in Table 2 are the only roots which are dominant, it follows by checking Table 2 that $L \cong A_l$ respectively C_l and correspondingly $\lambda - \sigma_0\lambda = \lambda_1 + \lambda_l$ respectively $2\lambda_1$. In the first case we get $\lambda = \lambda_1$ respectively λ_l. In the second case we get $\lambda = \lambda_1$. Therefore V is either the natural L-module $V(\lambda_1)$ or its contragredient. Since the adjoint L-module is $V(\lambda_1 + \lambda_l)$ or $V(2\lambda_1)$ respectively (see Table 2), it follows from Table 3 that there is no nonzero L-module morphism from $\wedge^m V$ to L, $m \in \underline{d}$, where d denotes the dimension of V (notice that $(\wedge^m V)^* \cong \wedge^m V^*$). Therefore there is no good triple constructed from the pairs L and V. We obtain a contradiction to the assumption. □

We proceed to prove C1.

Since $v^+ \wedge v^-$ is a generator of the L-module $V \wedge V$ (see Lemma A10), it follows that $v^+ \wedge v^- \wedge \underbrace{V \wedge \cdots \wedge V}_{n-3}$ generates $\wedge^{n-1}V$. Since ad $\neq 0$ we can have $\mathrm{ad}(v^+ \wedge v^-) \neq 0$ if $n = 3$ and $\mathrm{ad}(v^+ \wedge v^- \wedge v_1 \wedge \cdots \wedge v_{n-3}) \neq 0$ for some $v_i \in V$, $i \in \underline{n-3}$, if $n > 3$. Define

$$H_0 := \begin{cases} H \cap \mathrm{ad}(v^+ \wedge v^- \wedge \underbrace{V \wedge \cdots \wedge V}_{n-3}) & n > 3 \\ H \cap \{\mathrm{ad}(v^+ \wedge v^-)\} & n = 3. \end{cases}$$

<u>**Lemma 3.3**</u>: *Let $n > 3$. Let $z = ad(v^+ \wedge v^- \wedge v_{\mu_1} \wedge \cdots \wedge v_{\mu_{n-3}}) \neq 0$, where $v_{\mu_i} \in V_{\mu_i}$, $\mu_i \in \Pi(\lambda)$, $i \in \underline{n-3}$. Let $\gamma = \lambda + \sigma_0\lambda + \sum_{i=1}^{n-3} \mu_i$. Then $z \in L_\gamma$. If $\gamma = 0$, then $z \in H_0$; if $\gamma \neq 0$, then $\{0\} \neq [z, L_{-\gamma}] \subseteq H_0$.*

Proof: It is clear by Lemma 3.1 that $z \in L_\gamma$. If $\gamma = 0$, then $z \in H_0$ because of $L_0 = H$. So let $\gamma \in \Phi^+$. For $\gamma \in \Phi^-$ we can proceed analogously. By multiplying v^+ by some scalar we can assume that $z = x_\gamma$. Then by the choice of x_γ, y_γ, h_γ and the minimality of v^-,

$$\begin{aligned} -h_\gamma &= [y_\gamma, x_\gamma] \\ &= \mathrm{ad}(y_\gamma.v^+ \wedge v^- \wedge v_{\mu_1} \wedge \cdots \wedge v_{\mu_{n-3}}) \\ &\quad + \sum_{i=1}^{n-3} \mathrm{ad}(v^+ \wedge v^- \wedge v_{\mu_1} \wedge \cdots \wedge v_{\mu_{i-1}} \wedge y_\gamma v_{\mu_i} \wedge v_{\mu_{i+1}} \wedge \cdots \wedge v_{\mu_{n-3}}). \end{aligned}$$

If $y_\gamma.v^+ = 0$, then h_γ is already an element of H_0 and we are done with the proof.

Suppose now that $y_\gamma.v^+ \neq 0$ from which we shall deduce a contradiction. By Proposition A9, $\langle \lambda, \gamma \rangle \neq 0$, and this yields $x_\gamma.y_\gamma.v^+ = [x_\gamma, y_\gamma].v^+ = h_\gamma.v^+ =$