for all $v_i \in V_1$, $i \in \underline{n-1}$. Conversely, if $(L_i, V_i, \mathrm{ad}_i)$, $i = 1,2$, are two triples and if $\gamma : L_1 \to L_2$ is a Lie algebra isomorphism and $\tau : V_1 \to V_2$ is a vector space isomorphism such that identity (3.1) holds, then τ is an isomorphism of the associated n-Lie algebras. Therefore we shall view two triples with the above property as equivalent. It remains to determine the triples (L, V, ad) up to isomorphism of L and the equivalence of V.

From now on let K be an algebraically closed field of characteristic 0. We will assume that all vector spaces appearing in the following are finite dimensional over K.

If V is a simple n-Lie algebra over K, then Theorem 1.1.3 shows that the Lie algebra $Inder(V)$ $(= Der(V))$ is semisimple and that V as an $Inder(V)$-module is faithful and irreducible. Moreover the $Inder(V)$-module morphism ad: $\wedge^{n-1}V \to Inder(V)$ is surjective. If (L, V, ad) is a triple such that

G1) *L is a nonzero semisimple Lie algebra over K,*

G2) *V is a faithful irreducible L-module over K,*

G3) *ad is a surjective L-module morphism from $\wedge^{n-1}V$ onto the adjoint module L such that the map $\times^n V \to V$, $(v_1, \cdots, v_n) \to \mathrm{ad}(v_1 \wedge \cdots \wedge v_{n-1}).v_n$ is alternating,*

then the corresponding n-Lie algebra is simple, since an ideal of it is also an L-submodule of V. Moreover the derivation algebra is isomorphic to L. In fact, if ρ is the representation of L in V, then ρ is an isomorphism from L to the derivation algebra. A triple with G1), G2) and G3) will be called a good triple. The problem of determining the simple n-Lie algebras over K can be translated into that of finding the good triples (L, V, ad).

For the following notations one compares Humphreys [7] or the appendix in this work. Let L be a semisimple Lie algebra, let H be a maximal toral subalgebra of L, Φ the root system of L relative to H and $L = H \oplus \oplus_{\alpha\in\Phi} L_\alpha$, the root space decomposition of L. Further let Δ be a base of Φ, $\prec$ the half ordering on H^* relative to Δ and Φ^+ (Φ^-) the subset of Φ of positive (negative) roots. Let $(\cdot,\cdot)$ be the nondegenerate symmetric bilinear form on H^* which comes from the Killing form of L and $\langle\mu,\nu\rangle := \frac{2(\mu,\nu)}{(\nu,\nu)}$, where $\mu,\nu \in H^*$ and $(\nu,\nu) \neq 0$. For an $\alpha \in \Phi^+$ choose $x_\alpha \in L_\alpha$, $y_\alpha \in L_{-\alpha}$, $h_\alpha \in H$ such that $[x_\alpha, y_\alpha] = h_\alpha$, $[h_\alpha, x_\alpha] = 2x_\alpha$, $[h_\alpha, y_\alpha] = -2y_\alpha$. Recall that for this choice we have $\mu(h_\alpha) = \langle\mu,\alpha\rangle$ for all $\mu \in H^*$.

Let V be a faithful irreducible L-module with maximal weight $\lambda \in \Lambda^+$. Denote by $\Pi(\lambda)$ the set of all its weights. Then $V = \oplus_{\mu\in\Pi(\lambda)} V_\mu$.

<u>**Lemma 3.1**</u>: *Let L and V be as above. Let τ be an L-module morphism from $\wedge^m V$ $(m \geq 1)$ into L. If $v_{\mu_i} \in V_{\mu_i}$ is a vector of weight μ_i for all $i \in \underline{m}$, then $\tau(v_{\mu_1} \wedge \cdots \wedge v_{\mu_m}) \in L_\gamma$, where $\gamma = \sum_{i=1}^m \mu_i$.*