

2) The vector space V/I is an n -Lie algebra relative to the product

$$[\pi v_1, \dots, \pi v_n] := \pi[v_1, \dots, v_n],$$

where $\pi : V \rightarrow V/I$ denotes the canonical map.

3) Let $\psi : V \rightarrow V'$ be an n -Lie algebra homomorphism.

(a) If I' is an ideal of V' , then $\psi^{-1}I'$ is an ideal of V , in particular, $\text{Ker}\psi$ is an ideal of V . Moreover ψ induces an isomorphism $\tilde{\psi} : V/\text{Ker}\psi \rightarrow \text{Im}\psi$. If I is any ideal of V included in $\text{Ker}\psi$, then there exists a unique homomorphism $\phi : V/I \rightarrow V'$ such that $\psi = \phi\pi$.

(b) If ψ is in addition surjective, then the image of any ideal of V under ψ is an ideal of V' .

4) If $J \subseteq I$, then I/J is an ideal of V/J and $(V/J)/(I/J)$ is naturally isomorphic to V/I .

5) $(I + J)/J \cong I/(I \cap J)$.

Definition: Let V be an n -Lie algebra over K . We call an endomorphism D of V a derivation if for all $v_i \in V$, $i \in \underline{n}$:

$$D[v_1, \dots, v_n] = \sum_{i=1}^n [v_1, \dots, v_{i-1}, Dv_i, v_{i+1}, \dots, v_n]. \quad (1.9)$$

In consequence of identity (1.2) each left multiplication is a derivation. We refer to an endomorphism D of V as an inner derivation if it may be written as a sum of some left multiplications. We denote by $\text{Der}(V)$ ($\text{Inder}(V)$) the set of all (inner) derivations of V . One can verify

Proposition 1.1.2: Relative to the Lie bracket $\text{Der}(V)$ is a Lie algebra and $\text{Inder}(V)$ is an ideal of $\text{Der}(V)$.

Let V be an arbitrary n -Lie algebra and let U_i , $i \in \underline{n-1}$, be subspaces of V . We denote by $\text{ad}(U_1, \dots, U_{n-1})$ the subspace of $\text{Inder}(V)$ spanned by the left multiplications $\text{ad}(u_1, \dots, u_{n-1})$, $u_i \in U_i$. If U_i , $i \in \underline{n-1}$, are ideals of V , then $\text{ad}(U_1, \dots, U_{n-1})$ is an ideal of $\text{Inder}(V)$.

Let V be an arbitrary n -Lie algebra. Then the Lie algebras $\text{Der}(V)$ and $\text{Inder}(V)$ operate in a natural way on V , so we have a representation of them on V , or equivalently, V is a $\text{Der}(V)$ - and an $\text{Inder}(V)$ -module. If I is an ideal of V , that is, $[I, V, \dots, V] \subseteq I$, then I is an $\text{Inder}(V)$ -submodule of V . Conversely if