Thus $I'(a) \leq 0$, i.e.

$$
\begin{aligned}
C \int_0^{1/a} t^s \left(1 + \log(at)^{s-1}\right) \frac{dt}{t} = C \frac{a^{-s}}{s^2} \\
&\geq - \int_0^{1/a} f(t) t^s \left(1 + \log(at)^{s-1}\right) \frac{dt}{t} \\
&\geq - \int_0^{\infty} f(t) t^s \left(1 + \log(at)^{s-1}\right) \frac{dt}{t} \\
&= \underbrace{-(1 + (s-1)\log a) Z(s) - (s-1) Z'(s)}_{s \neq Z(s)}
\end{aligned}
$$

Taking

$$
\log a = -\frac{d}{ds} \log s(s-1) Z(s) = -\frac{1}{s} - \frac{1}{s-1} - \frac{Z'}{Z}(s)
$$

gives the desired inequality. □

A variant of the last lemma may be applied to the kissing number problem on replacing the exponential function in the definition of the theta series by a suitable Bessel function. More precisely, one has

$$
t \sum_{x \in L} F(t^{1/n}) = C \sum_{x \in L^*} \max\left(0, (1 - t^{-2/n} x^2)^p\right),
$$

$$
\left(F(t) = J_{q-1}(2\pi t)/t^{q-1},\ q = \frac{n}{2} + p + 1\right)
$$

with a suitable constant C. Here the Mellin transform of the left hand side is of the form

$$
\frac{\Gamma(q)\Gamma\left(\frac{ns}{2}\right)}{\Gamma\left(q - \frac{ns}{2}\right)} \sum_{\substack{x \in L \\ c \neq 0}} (x^2)^{-s}.
$$

Applying the last lemma (which, however cannot be applied literally, since $f(t)$ is not always nonnegative) gives than

Theorem.

$$
1.359^{n(1+o(1))}.
$$

Co-volumes

The following theorem is known: simply conn. Lie group

Theorem. *(Kajdan-Margoulis) Let G be a locally compact group without compact factors, and let μ be a Haar measure on G. Then there is a constant c_μ such that*

$$
\mu(G/\Gamma) \geq c_\mu
$$

for all discrete subgroups Γ of G.