Here μ is any Haar measure on $E_{\mathbb{R}}$. However, a natural choice is the ususal Lebesgue measure on E_R, and then one obtains

$$\mu(E_{\mathbb{R}}/E) = \mathrm{Reg}(L/K).$$

One can now proceed as with the kissing numbers to derive a lower bound for the relative regulator. Indeed, note that the left hand side is an increasing function. However, the summands here are much more complicated the the exponential function, and their study is quite cumbersome. Nevertheless, the procedure for kissing numbers, i.e. taking derivatives, dropping all terms with $\alpha \neq 0, 1$ for t large enough, and maximising in t can pushed through and gives

Theorem. *(Friedman-Skoruppa 97) One has*

$$\mathrm{Reg}(L/K) \geq \left(d_0\, d_1^{[L:K]}\right)^{[K:\mathbb{Q}]} \qquad (d_0 = (0.1/1.15)^{39},\ d_1 = 1.15)$$

$\gtrsim d_1^{(1-\varepsilon)[k:\mathbb{Q}]}$

Thus roughly spoken, the conjecture of Martinet-Bergé is true for $[L : K] >> 0$.

For $K = \mathbb{Q}$, i.e. the nonrelative case one can do much better since one can apply Mellin transformation (for $K = \mathbb{Q}$ the desired Dirichlet series does not converge). For this denote the left hand side of our last Poisson formula by

$$u\left(\frac{\mathrm{Reg}(L)}{\#E_{\mathrm{tor}}} + f(u)\right).$$

with $u = t^{n/2}$. It is well-known (this is Hecke's proof of the functional equation of partial zeta functions) that, with a suitable choice of μ, one has

$$D^*(s) := \int_0^\infty f(t)t^s\,\frac{dt}{t} = C^s\Gamma(s/2)^{r_1}\Gamma(s)^{r_2}\sum_{\substack{\in\mathbb{Z}_L\\ \alpha\neq 0}} N_{L/\mathbb{Q}}(\alpha)^{-s} =: C^s\gamma(s)D(s)$$

and

$$\operatorname*{Res}_{s=0} D^*(s) = -\frac{2^{r_1}R}{w}$$

with a suitable positive constant C. Set

$$g(s) := \log s(s-1)D^*(s).$$

note that

$$g(0) = \ln \frac{2^{r_1}R}{w}.$$