with asuitable positive integer n (Chinburg). It is easy to prove that

$$\frac{\operatorname{Reg}(L)}{\operatorname{Reg}(K)} \geq \operatorname{Reg}(L/K).$$

For the regulator of an arbitrary number field it is known:

Theorem. *(Zimmert 1981)* $\dfrac{\operatorname{Reg}(L)}{w_L} \geq .02 \cdot \exp(.46r_1 + .1r_2)$

Here r_1 and r_2 denote the real and complex places of $\cancel{K}$. L.

Moreover it is conjectured *(Conjecture Martinet-Bergé): There are absolute constants $C_1 > 0$ and $C_2 > 1$ such that*

$$\operatorname{Reg}(L/K) \geq C_0 C_1^{[L:K]}.$$

One can derive such bounds by slightly generalizing the method used for the kissing numbers. For this consider $E_{\mathbb{R}} = E \otimes \mathbb{R}$ as submodule of $\mathbb{R}_+^{P_L}$ via the embedding

$$E_{\mathbb{R}} = \cancel{E \otimes \mathbb{R}}_+^{P_L}, \quad \varepsilon \otimes 1 \mapsto \{|\varepsilon|_v\}.$$

$E \times \mathbb{R} \hookrightarrow \mathbb{R}_+^{P_L}$

For $\alpha \in K$ and $x \in \mathbb{R}_+^{P_L}$ let

$$\sigma(\alpha, x) = \sum_{v \in P_L} e_v |\alpha|_v^2 x_v^2.$$

Note that

$$\sigma(\varepsilon\alpha, x) = \sigma(\alpha, (\varepsilon \otimes 1)x)$$

for all $\varepsilon \in E$. Usual Poisson summation gives

$$t^{n/2} \prod_{v \in P_L} x_v^{e_v} \sum_{\alpha \in \mathbb{Z}_L} \exp(-\pi t \sigma(a, x)) = |D_L|^{-1/2} \sum_{\alpha \in \vartheta_L^{-1}} \exp(-\pi t^{-1} \sigma(a, x^{-1}))$$

D_L = disc.

with ~~suitabel psoitive constants~~ $\cancel{c_1, c_2 >}$ 0, whose exact description is not important at this point. Here $x \in \mathbb{R}_+^{P_L}$ and $t > 0$, and $n = [L : \mathbb{Q}]$. Note that both sides, as functions of x, are invariant under E. Hence, by integrating over a fundamantal domain $E_{\mathbb{R}}/E$ and the ususal trick of "unfolding" the integral, we obtain

$$t^{n/2} \frac{\mu(E_{\mathbb{R}}/E)}{\# E_{\text{tor}}} + t^{n/2} \sum_{\substack{\alpha \in \mathbb{Z}_L /E \\ \alpha \neq 0}} \int_{E_{\mathbb{R}}} \exp(-\pi t \sigma(a, x))\, d\mu(x)$$

$$= |D_L|^{-1/2} \frac{\mu(E_{\mathbb{R}}/E)}{\# E_{\text{tor}}} + |D_L|^{-1/2} \sum_{\substack{\alpha \in \vartheta_L^{-1} /E \\ \alpha \neq 0}} \int_{E_{\mathbb{R}}} \exp(-\pi t^{-1} \sigma(a, x)), d\mu(x).$$