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In this talk I would like to show how one can apply rather elementary means of the theory of modular forms to deduce absolute bounds for various algebro-geometric objects. The elmentary methods comprise merely Poisson summation, intgeration over lcally compcat groups, some manipulation of higher transcendental functions, and Mellin transforms, and they could be presented to first year graduate students. On the other hand the results are surprisingly non-trivial.

The first kind of object where I would like to illustrate the method, and maybe the easist application, are kissing numbers.

Kissing numbers

τ_n Recall the definition of the kissing number of dimension n: τ_n is the maximal number of balls of given radius 1 in euclidean n-space which can be placed around and touching another one without any two overlapping.

We shall consider here the lattice kissing number λ_n, which is defined as λ_n, but with the restriction that all balls belong to one and the same lattice. Thus

λ_n

$$\lambda_n = \max\{a(L) \,:\, L \text{ a lattice in euclidean } n\text{-space}\},$$

where $a(L)$ is the number of vectors of minimal positive length in L. The kissing numbers (of both kinds) are known only for a finite number of dimensions.

n	1	2	3	4	5	6	7	8	9	24
λ_n	2	6	12	@						196 560

Morover it it known (Kabatiansky-Levenshtein @):

$$\tau_n \leq 1.32042^{n(1+o(1))}.$$

The original proof of K-L is quite complicated. But here is a short proof which shows how one can obtain such a bound rather easily, at least for lattice kissing numbers.