Theorem (Friedman/Skoruppa 98):

Let L/K be an extension of number fields, $\mathcal{A}$ an ideal of L, let A_K the set of archimedean places of K, and let

$$E := \{\varepsilon \in O_L^* \,|\, \mathrm{N}_{L/K}(\varepsilon) \text{ root of unity of } K\}.$$

Then, for all *sufficiently good* $f : \mathbf{R}_{>0}^{A_K} \to \mathbf{C}$ one has

$$\sum_{\substack{\alpha\in\mathcal{A}/E \\ \alpha\neq 0|}} f\Big(\big\{|\mathrm{N}_{L/K}(\alpha)|_v\big\}_v\Big)$$

$$= \frac{2^{r_1}(2\pi)^{r_2}\mathrm{Reg}(L/K)}{\sqrt{D_{\mathcal{A}}}w_L} \int_{\mathbf{R}_{>0}^{A_K}} f(y)\,dy + \sum_{\substack{\alpha\in\widehat{\mathcal{A}}/E \\ \alpha\neq 0}} \widetilde{f}\Big(\big\{|\mathrm{N}_{L/K}(\alpha)|_v\big\}_v\Big),$$

where p_v, q_v is the number of real and complex places of L extending v, and

$$\widetilde{f}(y) = \pi^{[L:\mathbf{Q}]} \int_{\mathbf{R}_{>0}^{A_K}} f(x) \prod_{v\in A_K} k_{p_v,q_v}(x_v y_v)\,dy,$$

$$k_{p,q}(t) = \cos(\pi t)^{*p} * J_0(4\pi\sqrt{t})^{*q} \quad (* \text{ is Mellin convolution}).$$

(Sufficiently good: $\mathrm{supp}(f) \subseteq [a,b]^{A_K}$ with $0 < a < b$)