– Already said: lower bound ~~for~~ relative regulators (4)
– Lower bounds for covol. of arithmetic subgroups
I indicate the idea (which ~~proved~~ [turned out] to be useful in a more subtle setting for rel. regulators):
Insert a $t>0$ (and drop $\frac{1}{\# \Gamma_x}$)

$$\left(\mu(G/\Gamma) + \sum F(t^{2/n} x)\right)\cdot t = \mu(G^*/\Gamma^*) + \sum \tilde{F}(t^{-2/n} y)$$

assume $F(ty)$, $\tilde{F}(ty) \downarrow$ $(t\to\infty)$, then (right hand side) $\uparrow$,
thus $\frac{d}{dt}$ (~~left~~ l.-s.) ≥ 0; i.e.

$$\mu(G/\Gamma) \geq \sum_{\substack{x\in L/\Gamma \\ x\neq 0}} -\left(F(t^{2/n}x) + \frac{2}{n}\, t^{\frac{2}{n}} F'(t^{\frac{2}{n}}x)\right)$$

if G acts transitively we may assume that $e=(1,0,\dots,0)\in L$, if F is sufficiently nice the summands ≥ 0, Hence

$$\mu(G/\Gamma) \geq \sup_{t\geq 0} -\left(F(te) + \frac{2}{n}\, t\, F'(te)\right)$$

for all F s.th. .....

To give finally a theorem (true, but not so useful in its momentaneous form):

Thm: L/k ext. of # fields, $\mathfrak{a}$ ideal of L,
$E = \{\varepsilon \in \mathcal{O}_L^* \mid N_{L/k}(\varepsilon) = \text{root of unity}\}$ (rel. units L/k)
A_k = archimed. places of L

$$\sum_{\substack{\alpha\in\mathfrak{a}/E \\ \alpha\neq 0}} f\left(\left(|N_{L/k}\alpha|_v\right)_v\right) = \frac{2^{r_2}}{\sqrt{d_{\mathfrak{a}}}} \sum_{\substack{\beta\in\hat{\mathfrak{a}}/E \\ \beta\neq 0}} \tilde{f}\left(\left(|N_{L/k}\beta|_v\right)_v\right)$$

$$+ \frac{2^{r_1}(2\pi)^{r_2}\,\mathrm{Reg}(L/k)}{\sqrt{d_{\mathfrak{a}}}\; w_L} \int_{\mathbb{R}_{>0}^{A_k}} f(y)\,dy$$

for $f:\mathbb{R}_{>0}^{A_k}\to\mathbb{C}$ compactly supported
where $\tilde{f}(y) = \pi^{[L:\mathbb{Q}]/2} \int_{\mathbb{R}_{>0}^{A_L}} f(x) \prod_{v\in A_k} k_{p_v,q_v}(x_v y_v)\,dy$
p_v, q_v = places of L over v
$k_{p,q} = k_{1,0}^{*p} * k_{0,1}^{*q}$ $\left(k_{1,0}(t) = \cos \pi t,\ k_{0,1}(t) = J_0(4\pi\sqrt{t})\right)$