Suppose

G locally compact, acting on $\mathbb{R}^n$, $L \subseteq \mathbb{R}^n$ lattice, ~~invt~~

$\Gamma \leq G$ discrete subgroup s.t. $\Gamma L \subseteq L$, μ = right-invariant Haar measure on G

Poisson: $$\sum_{x \in L} f(gx) = c \cdot \sum_{y \in L^*} \hat{f}(g^* y) \quad (g \in G,\ g^* \sim \text{dual action})$$

Both sides are Γ-invariant (right side with respect to dual action)

Integrate over G/Γ: $\int_{G/\Gamma} d\mu(g)$

Apply the usual trick of unfolding the integral (let me give some details for pointing out the difficulties)

$$\int_{G/\Gamma} \sum_{x \in L} f(gx)\, d\mu(g)$$

$$= \int_{G/\Gamma} \sum_{\gamma \in \Gamma} \sum_{\substack{x \in L/\Gamma \\ x \neq 0}} \frac{1}{\#\Gamma_x} f(g\gamma x) \quad \left(\begin{array}{l} 1^{st} \text{ problem:} \\ - x \to \#\Gamma_x \ ? \\ - \Gamma_x \text{ finite } ? \end{array}\right)$$

$$+ \mu(G/\Gamma)\, f(0)$$

(impose conditions on G action and on f)

$$= f(0)\,\mu(G/\Gamma) + \sum_{\substack{x \in L/\Gamma \\ x \neq 0}} \frac{1}{\#\Gamma_x} \int_G f(gx)\, d\mu(g)$$

hence one obtains (if everything works well):

$$f(0)\mu(G/\Gamma) + \sum \frac{1}{\#\Gamma_x} F(x)$$

$$= \hat{C}(y)(f)\, \mu(G/\Gamma) + C_2 \sum_{x \in L^*/\Gamma^*} \frac{1}{|\Gamma^*_y|} \tilde{F}(y)$$

<u>Problems:</u>

- when $F(x) = \int f(gx)\, d\mu(g)$?
- ~~write~~ $\tilde{F}(y)$ as integral transform of F (not of f).