taking <u>inverse Mellin transforms</u>:

$$\text{cst} + \sum_{\substack{x \in \mathfrak{a}/\mathcal{O}^* \\ x \neq 0}} f(x) \sim \text{cst} + \sum_{\substack{x \in \mathfrak{a}^{-1}\mathfrak{d}_k^{-1}/E \\ x \neq 0}} \hat{f}(x)$$

$$\left(\mathfrak{a} \overset{\text{view}}{\hookrightarrow} k \otimes_{\mathbb{Q}} \mathbb{R} \subseteq \mathbb{R}^{r_1 + 2r_2} \right)$$

Before I say something about how to produce a ~~more~~ general formula here some ~~indications~~ comments why this could be interesting:

- $\mathrm{Res}_{s=1}\, \zeta^*(\mathfrak{C}, s) \sim \mathrm{Reg}(k)$
  (Zimmert used for: $\mathrm{Reg}(k) > \left(\begin{smallmatrix}\text{absolute}\\ \text{const}\end{smallmatrix} > 0\right)$ for all k)
  One wants for <u>relative extension L/k</u> also
  $\zeta_{L/k}(\mathfrak{C}, s)$ such that $\mathrm{Res}_{s=1} \sim$ rel. reg. of L/k
  Friedman-An: used this approach for: Rel. reg. > absolute constant if $L/k \gg 0$
  $\zeta_{L/k}(\ ) = ?$ but on the inverse Mellin side it exists (conj. of Bergé-Martinet)
  however only for <u>special choice</u> of f; one could <u>probably do better</u> <u>with more general</u> f?

- useful for proving <u>absolute lower bounds</u> for <u>covolume</u> <u>of arithmetic subgroups</u> of algebraic groups (details in a moment). (Kazdan-Margulis: G semi-simple Lie without compact factors $\Rightarrow \exists\, c > 0$: $\Gamma \underset{\text{discr}}{\leq} G \Rightarrow \mu(G/\Gamma) \geq c$)

<u>How</u> can one <u>prove</u> a generalized Poisson summation formula?
I only know one approach: