Everybody knows (and probably already used) Poisson summation:

$$\mathrm{vol}(\mathbb{R}^n/L)^{1/2} \sum_{x\in L} f(x) = \mathrm{vol}(\mathbb{R}^n/L^*)^{1/2} \sum_{y\in L^*} \hat{f}(y)$$

($L \le \mathbb{R}^n$ lattice, $L^* = \{y \in \mathbb{R}^n \mid \langle y, L\rangle \subseteq \mathbb{Z}\}$ dual lattice, $f: \mathbb{R}^n \to \mathbb{C}$ "strong")

very often one has the situation of a group G acting on $\mathbb{R}^n$ with $\underset{\text{subgroup}}{\Gamma \le G}$, $\Gamma L \le L$, $f: \mathbb{R}^n \to \mathbb{C}$ G-invariant

Here one might expect a generalized Poisson Summation formula

$$\sum_{x\in L/\Gamma} f(x) \sim \sum_{y\in L^*/\Gamma^*} \tilde{f}(y)$$

($*$: dual action of G: $\langle g^{-1}x, y\rangle = \langle x, g^* y\rangle$,
$L^*/\Gamma^* = L^*$ mod dual action of Γ,
$f: \mathbb{R}^n/G \to \mathbb{C}$, $\tilde{f}$ = suitable (??) integral transform)

Examples are:

- $G = O(n)$, $f: \mathbb{R}^n \to \mathbb{C}$ radially invariant, $\tilde{f}$ = Hankel transform } trivial (since Γ finite)

- another one provided by Hecke's proof of functional equation for partial zeta functions

$$\zeta^*(C,s) = \Big(\sum_{\mathfrak{a}\in C} N(\mathfrak{a})^{-s}\Big)\, \Gamma(\tfrac{s}{2})^{r_1}\, \Gamma(s)^{r_2} \sim \zeta^*(C^*, 1-s)$$

(C ideal class in a $\overset{\text{integral}}{\#}$ field K, r_1, r_2 = # embeddings...)
$C^* = [\mathfrak{a}^{-1}\mathfrak{d}^{-1}]$ if $C = [\mathfrak{a}]$)