(4)

This also gives rise to improvements:

$$\vartheta_L^{mod.}(t) = \sum_{x \in L} j(|x| t^{1/n}) \quad , \ j \text{ some good function (say } j(0)=1) \quad j : \mathbb{R}_{\geq 0} \to \mathbb{R}$$

"Poisson"-sum: $d(L)^{-1} \sum_{x \in L^\#} \hat{j}(|y| / t^{1/n}) = t\, \vartheta_L^{mod}(t) =: t(1+f(t))$

where $\hat{j}(l) = \int_0^\infty da\, j(a) \cdot J_{\frac{n-2}{2}}(2\pi a l) a^{n/2}$ "Hankel transform"

if $\hat{j}$ decreasing, non-negative then t. m. gives

$$a_1(L) \leq \frac{1}{\frac{n}{2} s(s-1)} \exp\left(1 + \frac{s}{s-1} + n\left[\frac{s}{2}\right.\right.$$

$$a_1(L) \leq \exp\left(n\left[\frac{s}{2} \frac{\gamma'}{\gamma}\left(\frac{ns}{2}\right) - \frac{1}{n} \log \gamma\left(\frac{ns}{2}\right)\right] + o(1)\right)$$

where $\gamma(s) = \int_0^\infty j(t)\, t^s \frac{dt}{t}$. $\quad$ cond.: $J_p(a)\, t^{-p}$

For the moment I applied this to ~~$j(t)$ = some Bessel function~~ :

~~and hence~~ $\gamma(s) = \cancel{\frac{\Gamma(2p)\,\Gamma(s)}{\Gamma(2p-s)}}$ ~~$1 < s < \frac{1}{2}$~~

$$\cancel{\lambda_n \leq \exp(s(1+\log .5)(n+o(1))) = 1.35914^{n(1+o(1))}}$$

$(s>1)$

~~This can be improved!~~

~~Also one should look at the concrete bounds for small n that one could obtain!~~

$$a_1(L) \leq \frac{\Gamma(p+1-\frac{ns}{2})}{\frac{n}{2}\Gamma(p+1)\Gamma(\frac{ns}{2})\, s(s-1)} \exp\left(1 + \frac{s}{s-1} + \frac{ns}{2}\left[\psi\left(\frac{ns}{2}\right) + \psi\left(p+1-\frac{ns}{2}\right)\right]\right)$$

for $\left(\underset{?}{\frac{3n}{2}} >\right) p > \left(\underset{?}{\frac{ns}{2}} - 1\right)$

Analysis as above

$$a_1(L) \leq \exp\left(s(1+\log .5)(n+o(1))\right) \underset{(s=1)}{=} 1.35914^{n(1+o(1))}$$