Thm ~~cost~~

Illegal argument — 1 corr. result:

$$G(s) = \log s(s-1) Z(s)$$ (OK near $s=0$ and $s>1$, ... $s-1$)

Think of $G(s)$ being convex, hence

$$G(0) \geq G(s) - s\,G'(s) + \text{"}>0\text{"}$$ ($s \to 0$)! (no sense in general; singular ... (0,1)!)

thus — taking exp. —

~~$\vartheta(s) \leq \frac{1}{\Gamma(\frac{ns}{2})} \frac{1}{\frac{n}{2}s(s-1)} \exp($~~

negative

$$a_1(L) \leq \vartheta_L(s) \leq \frac{1}{\frac{n}{2}s(s-1)} \frac{1}{\Gamma(\frac{ns}{2})} \exp\left(1 + \frac{s}{s-1} + \frac{n}{2} s\,\psi\left(\frac{ns}{2}\right) + s\frac{\vartheta_L'(s)}{\vartheta_L(s)}\right)$$

Asymptotically

$$a_1(L) \leq \exp\left(n\left[\psi\left(\frac{ns}{2}\right) - \frac{1}{n}\log\Gamma\left(\frac{ns}{2}\right) + O(\ldots)\right]\right)$$

Using $\log\Gamma(s) = (s-\frac{1}{2})\log s - s + \log\sqrt{2\pi} + \frac{1}{12s} + O(s^{-2})$

$$\psi(s) = \log s - \frac{1}{2s} + O(s^{-2})$$

you obtain

$$a_1(L) \leq \exp(.5s(n+O(1)))$$ ... + better.

Can give correct argument:

Thm (n) Let $f : \mathbb{R}_{>0} \to \mathbb{R}$ measurable, non-negative such that $(1+f(t))t$ is mon. increasing.

Assume $Z(s) = \int_0^\infty f(t)\, t^s \frac{dt}{t}$ conv. for $s>1$.

Then

$$Z(s) \leq \frac{1}{s(s-1)} \exp\left(1 + \frac{s}{s-1} + s\frac{Z'(s)}{Z(s)}\right) \quad (s>1).$$

(Apply to $1+f(t) = 1+a(t^{2/n})$.)