Regulator:

$|\cdot|_1, \ldots, |\cdot|_r, |\cdot|_{r+1}$ unendliche Primstellen von k

$\varepsilon_1, \ldots, \varepsilon_r$ Fund. einh., $e_j = \begin{cases} 1 & |\cdot|_j \text{ reell} \\ 2 & |\cdot|_j \text{ komplex} \end{cases}$

$$\mathrm{Reg}(k) = \left| \det \left(\log |\varepsilon_i|_j^{e_j} \right)_{1 \le i,j \le r} \right|$$

Problem: $\mathrm{Reg}(k) > \ldots ? \ldots$

Beispiel: K reell quadratisch $(r=1)$

$$\left\{ \begin{matrix} \text{reell quadr.} \\ \text{Einheiten} \end{matrix} \right\} = \left\{ \begin{matrix} \text{Lösungen von} \\ x^2 - tx \pm 1 = 0, \end{matrix} \quad t^2 - 4(\pm 1) \neq 0 \text{ in } \mathbb{Z} \right\}$$

$$= \left\{ \frac{t \pm \sqrt{t^2-4}}{2}, \frac{\cancel{t} \pm \sqrt{t^2+4}}{2} \right\}$$

kleinste Positive: $\varepsilon = \dfrac{-1+\sqrt{5}}{2}$ (gold. Schnitt)

$$\mathcal{O}^*_{\mathbb{Q}(\sqrt{5})} = \pm \varepsilon^{\mathbb{Z}}$$

Fazit: $\mathrm{Reg}(k \text{ reell quadr.}) > \log \left| \dfrac{-1+\sqrt{5}}{2} \right| = 0{,}481\ldots$

<u>Remak</u> (37, 52):

$\exists\, c_0 > 0, c_1 > 1$: $\mathrm{Reg}(k \text{ total reell}) > c_0 c_1^{r_k}$ $\quad (c_1 = 1{,}012)$

$\exists\, c_2, c_3 > 0$: $\mathrm{Reg}(k \text{ beliebig}) > \left(\dfrac{c_2}{[k:\mathbb{Q}]} \right)^{c_3 [k:\mathbb{Q}]}$

$(\to 0$ für $[k:\mathbb{Q}] \to \infty)$

<u>Pohst</u> (77): $\mathrm{Reg}(k \text{ total reell}) \geqq \log \left| \dfrac{-1+\sqrt{5}}{2} \right|$ $\quad (r_1 \neq 6, 8, 10)$

<u>Zimmert</u> (80): $\dfrac{\mathrm{Reg}(k \text{ bel.})}{\# W_k} \geqq 0{,}02 \exp(0{,}46 r_1 + 0{,}1 r_2)$

$\left(\geqq \dfrac{0{,}02}{\exp(0{,}1)} \exp(0{,}1(r-1)) \right) \geq 0{,}018 \cdot (1{,}1)^{r_k}$

(W_k = # Einheitswurzeln in k)

und $\mathrm{Reg}(k \text{ bel.}) \geqq 0{,}056$