

Table 1: Data of certain bosonic \mathcal{W} -algebras related to the ADE classification

\mathcal{W} -algebra	$H_{c(p,q)}$
$\mathcal{W}(2)$ p, q prime	$\{h(p, q, r, s) \mid r = 1, \dots, q-1, (2, r) = 1,$ $s = 1, \dots, p-1\}$
$\mathcal{W}(2, \frac{(m-1)(q-2)}{2})$ $p = 2m$ q, m prime	$\{h(p, q, r, s) \mid r = 1, \dots, (q-1)/2,$ $s = 1, \dots, m, (2, s) = 1\}$
$\mathcal{W}(2, q-3)$ $p = 12, q \geq 5$ q prime	$\{\min(h(p, q, r, 1), h(p, q, r, 7)) \mid r = 1, \dots, (q-1)/2\} \cup$ $\{\min(h(p, q, r, 5), h(p, q, r, 11)) \mid r = 1, \dots, (q-1)/2\} \cup$ $\{h(p, q, r, 4) \mid r = 1, \dots, (q-1)/2\}$
$\mathcal{W}(2, q-5)$ $p = 30, q \geq 7$ q prime	$\{\min(h(p, q, r, 1), h(p, q, r, 11)) \mid r = 1, \dots, (q-1)/2\} \cup$ $\{\min(h(p, q, r, 7), h(p, q, r, 13)) \mid r = 1, \dots, (q-1)/2\}$

$$c = c(p, q) = 1 - 6 \frac{(p-q)^2}{pq}, \quad h(p, q, r, s) = \frac{(rp-sq)^2 - (p-q)^2}{4pq}$$

Table 2: Data of the six rational models with irreducible $SL(2, \mathbb{Z})$ -representation

\mathcal{W} -algebra	c	\tilde{c}	H_c
$\mathcal{W}_{G_2}(2, 1^{14})$	$-\frac{8}{5}$	$\frac{16}{5}$	$\frac{1}{5}\{0, -1, 1, 2\}$
$\mathcal{W}_{F_4}(2, 1^{26})$	$\frac{4}{5}$	$\frac{28}{5}$	$\frac{1}{5}\{0, -1, 2, 3\}$
$\mathcal{W}(2, 4)$	$-\frac{444}{11}$	$\frac{12}{11}$	$-\frac{1}{11}\{0, 9, 10, 12, 14, 15, 16, 17, 18, 19\}$
$\mathcal{W}(2, 6)$	$-\frac{1420}{17}$	$\frac{20}{17}$	$-\frac{1}{17}\{0, 27, 30, 37, 39, 46, 48, 49, 50,$ $52, 53, 55, 57, 58, 59, 60\}$
$\mathcal{W}(2, 8)$	$-\frac{3164}{23}$	$\frac{28}{23}$	$-\frac{1}{23}\{0, 54, 67, 81, 91, 94, 98, 103, 111,$ $112, 116, 118, 119, 120, 122, 124,$ $125, 129, 130, 131, 132, 133\}$
\mathcal{WC}_3 ($\mathcal{W}(2, 4, 6)$)	$-\frac{13}{15}$	$\frac{17}{15}$	$\frac{1}{180}\{0, -15, -8, -3, 12, 37, 57, 60, 100,$ $117, 120, 132, 145, 252, 285, 405\}$

(Product Formulas). Let c be one of values $-444/11, -1420/17, -3164/23$, and let l denote its denominator. For any integer x , relatively prime to l , set

$$[x]_l = q^{l/24 - (l-2x_0)^2/8l} \prod_{\substack{n \geq 1 \\ n \equiv \pm x \pmod{l}}} (1 - q^n)^{-1},$$

where x_0 is that integer which satisfies $1 \leq x_0 \leq (l-1)/2$, $x_0 \equiv \pm x \pmod{l}$. Then the set of functions $\xi_{c,h}$ ($h \in H_c$) equals

$$\left\{ \begin{array}{ll} \left\{ \frac{\eta(q^l)}{\eta(q)} \frac{[4x]_l [5x]_l}{[x]_l}, \frac{\eta(q^l)}{\eta(q)} [x]_l \right\}_{x=1, \dots, (l-1)/2} & \text{if } l = 11 \\ \left\{ \frac{\eta(q^l)}{\eta(q)} \frac{[6x]_l [7x]_l [8x]_l}{[x]_l}, \frac{\eta(q^l)}{\eta(q)} [x]_l [5x]_l \right\}_{x=1, \dots, (l-1)/2} & \text{if } l = 17 \\ \left\{ \frac{\eta(q^l)}{\eta(q)} \frac{[8x]_l [9x]_l [10x]_l [11x]_l}{[x]_l}, \frac{\eta(q^l)}{\eta(q)} [x]_l [5x]_l [7x]_l \right\}_{x=1, \dots, (l-1)/2} & \text{if } l = 23. \end{array} \right.$$