To construct elements of $M_k(\rho)$ for a given ρ the following observation is useful:

<u>Theorem</u> [Kloosterman, Wolfart-Nobs]:
Every irred. rep. of Γ whose kernel is a cong. subgroup is obtained as subrep. of a "Weil-representation".

<u>Weil representation associated to (M, Q)</u>

- M finite abelian group
- $Q: M \to \mathbb{Q}/\mathbb{Z}$ quadratic form $\left(\begin{array}{l}\text{i.e. } B(x,y) = Q(x+y)-Q(x)-Q(y) \ \mathbb{Z}\text{-bilin.} \\ + \ Q(x) = Q(-x)\end{array}\right)$

Γ acts on $\mathbb{C}^M$:

$(f \mid T)(x) = e^{2\pi i Q(x)} f(x)$

$f \mid S$ = finite Fourier transform of f w.r.t. $e^{2\pi i B(x,y)}$

Given $\rho: \Gamma \to Gl(n, \mathbb{C})$ (irreducible ...)

choose $\rho \subseteq$ ~~rep. of Γ on~~ Weil representation a.t. (M, Q),
compute $\phi: M \to \mathbb{C}^n$ s.t. $\phi \mid A = \rho(A)\phi$
choose $\underbrace{\text{lattice}}_{\text{even integral}}$ $L \subseteq \mathbb{R}^{2k}$ s.t. $(L^*/L, \frac{x^2}{2} \bmod \mathbb{Z}) \cong (M, Q)$
choose spherical poly. $p^{(x_1, \ldots, x_{2r})}$ (i.e. homog. + $\Delta p \equiv 0$)

<u>Theorem</u> [everybody since 19th cent]

$$\mathbb{C}^M \ni f \longmapsto \Theta_f = \sum_{x \in L^*} f(x)\, p(x)\, q^{x^2/2} \quad \text{is a } \Gamma\text{-homomorphism}$$

$$\left(\text{i.e. } \Theta_f \big|_{r+\deg p} A = \Theta_{f \mid A}\right)$$

<u>Corollary</u>: $\sum_{x \in L^*} \phi(x)\, p(x)\, q^{x^2/2} \in M_{r+\deg p}(\rho)$.