Theorem [~]

$k \in \frac{1}{2}\mathbb{Z}$, $\rho : \tilde{\Gamma} \longrightarrow GL(n, \mathbb{C})$ rep. with finite image and

$\rho((\pm id, \varepsilon)) = \bar{\varepsilon}^{2k}\, id$ for all $(\pm id, \varepsilon) \in \tilde{\Gamma}$.

Then

$$\dim M_k(\rho) - \dim S_{2-k}(\bar{\rho}) = \frac{k-1}{12} \cdot n$$
$$+ \frac{1}{4} \mathrm{Re}\left(e^{\pi i k/2}\, \mathrm{tr}\, \rho(S, \sqrt{\tau})\right)$$
$$+ \frac{2}{3\sqrt{3}} \mathrm{Re}\left(e^{\pi i (2k+1)/6}\, \mathrm{tr}\, \rho(ST, \sqrt{\tau+1})\right)$$
$$+ \frac{1}{2} a(\rho) - \sum_{j=1}^{n} \mathbb{B}_1(\lambda_j).$$

Here $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$,

λ_j $(1 \le j \le n)$ s.th. $e^{2\pi i \lambda_j}$ runs thru eigenval. of $\rho(T)$,

$a(\rho) = \#\, j$ s.th. $e^{2\pi i \lambda_j} = 1$

$\mathbb{B}_1(x) = x' - \frac{1}{2}$ if $x \in x' + \mathbb{Z}$, $0 < x' < 1$, $= 0$ ~~otherw.~~ if $x \in \mathbb{Z}$.

$S_{..}(\rho)$ def. like $M_{\_}(\rho)$ but with big O repl. by small o.

How to obtain a basis for $M_k(\rho)$?

- ~~Answer~~ useful for
  - obtaining explicit formulas for conf. char.
  - if dim-comput. do not suffice to conclude uniqueness