① Oberwolfach, Juni 94

Methods for calculating explicit formulas for conformal characters of W-algebras

W-algebras:

Key-words:
- arises in 2-dim CFT
- special vertex-oper. algebra
- rel. to Kac-Moody algebras

$W \approx (\mathcal{F}, \omega_0)$ where

- $\mathcal{F}$ (super) Lie algebra extend. Virasoro algebra $\mathcal{V}$

  ($\mathcal{V} = \mathbb{C}[\ldots L_{-1}, L_0, L_{+1}, \ldots; C]$

  $[L_m, L_n] = (n-m) L_{m+n} + \frac{n^3-n}{12} \delta_{m,-n} C$

  $[L_n, C] = 0$)

  satisfying certain axioms (analyticity cond. on $[\,,\,]$)
- in partic. $[C, \mathcal{F}] = 0$
- $\omega_0 : \mathcal{F} \longrightarrow \mathrm{End}(H_{\omega_0})$ a "lowest-weight repr"

  (vacuum rep)

  satisfy. cert. axioms

  in partic.: ω_0 irred., $\omega_0(C) = c \cdot$ identity

  c = central charge of W

  ~~H_{ω_0}~~ =

lowest-weight rep. ω of W: $\omega : \mathcal{F} \longrightarrow \mathrm{End}(H_\omega)$ Lie-algebra rep.

+ cert. axioms

in partic.: $\mathcal{H}_\omega = \bigoplus_{n \geq 0}^{\infty} H_\omega^n$

s.t. $H_\omega^n = \omega(L_0)$-eigenspace of eigenvalue $h+n$

some fixed $h \in \mathbb{R}$ (indep. of n), $\dim \mathcal{H}_\omega^n < \infty$

h = conformal dimension of ω

$$\chi_\omega = \mathrm{tr}\left(q^{L_0 - \frac{c}{24}}, H_\omega\right) = q^{h - \frac{c}{24}} \sum_{n=0}^{\infty} (\dim H_\omega^n)\, q^n \in \mathbb{C}[[q]]$$

conformal character of ω