(11)

As a Corollary one obtains

4. Application to modular forms

~~Proposition~~ For $\Delta \in Sl_2(\mathbb{Z})$, $\Delta_i, r_i \in \mathbb{Z}$, $\Delta_i \equiv r_i^2 \bmod 4m$, Δ_i fund. define

$$f(\tau) := f_{A,\Delta_0,r_0,\Delta_1,r_1}(\tau) = \sum_{\ell=1}^{\infty} \left(\sum_{a \mid \ell} \left(\frac{\Delta_0}{a}\right) \left\{ \cancel{\sum_{Q \in \mathcal{L}(\Delta\Delta_1 \frac{\ell^2}{a^2}, r_0 r_1 \frac{\ell}{a})} \chi_{\Delta_0}(Q) (\mathrm{sign}(Q \circ A) + 0} \right. \right.$$

$$\left. + \, C_{A,\Delta_0,r_0}\left(\Delta_1 \frac{\ell^2}{a^2}, r_1 \frac{\ell}{a}\right) \right\}$$

Then this defines a cusp form in $\mathfrak{M}_2(m)$ if $\Delta_0 \neq 0$, and – up to ~~the addition of a~~ a suitable constant term – a modular form in $\mathfrak{M}_2(m)$. Any element in $\mathfrak{M}_2^{cusp}(m)$ can be ~~obtained as~~ written as a linear combination of the $f_{A,\ldots}$.

~~Example $S_2(\Gamma_0(11)) = 1$-dimensional~~

~~Computation~~ of the Fourier development of Φ_{A,Δ_0,r_0} –

$$r_A^5(\vartheta_{\Delta_0,r_0}(\tau,z)) = \sum_{\substack{\Delta, r \\ r^2 \equiv \Delta \bmod 4m}} e^{2\pi i\left(\frac{r^2-\Delta}{4m} u + \frac{r^2+\Delta}{4m} iv + rz\right)} \, \cancel{C_{\Delta_0,r_0}}(\Delta, r)$$

$$C(\Delta,r) = \int_0^{i\infty} \left\{ C_r(\Delta, r; At) \, dAt - \sigma C_r(\Delta, r, gAg) \, dgAgt \right\}$$

$$= \int_0^{i\infty} \left(\sum_{Q \in \mathcal{L}(\Delta\Delta_0, r r_0)} \chi_{\Delta_0}(Q) \frac{Q \circ A(t)}{y^2} e^{-\lambda \frac{\widehat{Q \circ A(t)}^2}{y^2}} \cancel{dA} \right.$$

$$\left. - \sigma \sum_{Q \in \mathcal{L}(\Delta\Delta_0, r r_0)} \chi_{\Delta_0}(Q) \frac{Q \circ g A_g(t)}{y^2} e^{-\lambda \frac{\widehat{Q \circ g A_g(t)}^2}{y^2}} \right) \overline{dA}$$

$\lambda = \frac{\pi v}{m |\Delta_0|}$

replace in the second sum $Q \to -Q \circ g$, use $\chi_{\Delta_0}(-Q \circ g) = -\chi_{\Delta_0}(Q)$