Theo

For $A \in Sl_2(\mathbb{Z})$, $\Delta_0, r_0 \in \mathbb{Z}$ such that $\Delta_0 \equiv r_0^2 \bmod 4m$ and Δ_0 = fundamental discriminant define

$$\phi_{A,\Delta_0,r_0} = \sum_{\substack{\Delta, r \in \mathbb{Z} \\ \Delta \equiv r^2 \bmod 4m}} C_{A,\Delta_0,r_0}(\Delta, r)\, e^{2\pi i\left(\frac{r^2-\Delta}{4m}u + \frac{r^2+|\Delta|}{4m} iv + rz\right)}$$

$$C_{A,\Delta_0,r_0}(\Delta, r) = \sum_{\substack{Q \in \mathcal{L}(\Delta_0,\Delta,r_0,r),\, a'c'<0}} \chi_{\Delta_0}(Q)\, \mathrm{sign}(a')$$

$$+ \sum_{\substack{Q \in \mathcal{L}(\Delta_0,\Delta,r_0,r) \\ a'=0,\ 0<c'<m|\Delta_0|}} \left(\frac{c'}{m|\Delta_0|} - \frac{1}{2}\right)$$

$$- \sum_{\substack{Q \in \mathcal{L}(\Delta_0,\Delta,r_0,r) \\ c'=0,\ 0<a'<m|\Delta_0|}} \left(\frac{a'}{m|\Delta_0|} - \frac{1}{2}\right)$$

notation: ($Q(At)(ct+d)^2 = a't^2 + b't + c'$)

Then $\phi_{A,\Delta_0,r_0} \in S_{2,m}^{\mathrm{sign}(\Delta_0)}$. Moreover any Jacobi form in $S^{\pm}_{2,m}$ is a linear comb of the functions ϕ_{A,Δ_0,r_0}.

Remark First sum is finite: there are only finitely many ~~a,b,c~~ $at^2+bt+c \in \mathbb{Z}[t]$ such that

$b^2 - 4ac = D$, $ac < 0$

($\Leftrightarrow |b| < \sqrt{D}$, and $|ac| = \frac{D-b^2}{4}$)