Now to construct Jacobi ~~forms~~ from $\vartheta_{\Delta_0, r_0}(\tau, z; t)$ we set

$$\phi(\tau,z) := \phi_{A, \Delta_0, r_0}(\tau, z) := r_A^{\sigma}\left(\overline{\vartheta_{\Delta_0, r_0}(\tau, z; \bullet)}\right) \qquad (\sigma = \operatorname{sign} \Delta_0).$$

Clearly

① $\phi(\tau,z)$ transforms like a Jacobi form of weight 2.

② Compute the Fourier expansion of ϕ. If it has the correct shape, then $\phi(\tau,z)$ is a Jacobi form.

③ ~~$\phi_{A; \Delta_0, r_0}$ = kernel funct for $r_A^{\sigma} \circ \mathcal{S}_{\Delta_0, r_0}$~~

~~If $\phi \in J^{\varepsilon}_{2,m}$ satisfies $\langle \phi \mid \phi_{A, \Delta_0, r_0} \rangle = 0 \ \forall A, \Delta_0, r_0 \ldots \operatorname{sign} \Delta_0 = \varepsilon$,~~

~~then $r_A^{\varepsilon}(\langle \phi \mid \vartheta_{\Delta_0, r_0}(\bullet,) \rangle$~~

③ $\phi_{A; \Delta_0, r_0}$ = Kernel function for $r_A^{\sigma} \circ \mathcal{S}_{\Delta_0, r_0}$

(i.e. $r_A^{\sigma} \circ \mathcal{S}_{\Delta_0, r_0} \phi = \langle \phi \mid \phi_{A, \Delta_0, r_0} \rangle$).

Thus: $\phi \perp \operatorname{span}_{\mathbb{C}} \langle \phi_{A, \Delta_0, r_0} \mid A, \Delta_0, r_0, \ldots \rangle$

$$\Rightarrow \phi \in \bigcap_{A, \Delta_0, r_0 \ldots} \operatorname{Ker} r_A^{\sigma} \circ \mathcal{S}_{\Delta_0, r_0} \underset{(r_A^{\sigma} \text{ injective!})}{\Longrightarrow} \phi \in \bigcap_{\Delta_0, r_0 \ldots} \operatorname{Ker} \mathcal{S}_{\Delta_0, r_0}$$

$\xRightarrow{\text{Main Theorem}} \phi = 0.$

~~Thus modulo~~ the comp

We shall compute the Fourier expansion of ϕ in a moment. The final result will then be