Proof Tedious application of the Poisson summation formula.
(cf. H. Cohen's lecture)

Examples

(1) $F > 0$, $p(X) = 1$, ~~∇^2~~ F $\det F = 1$ $\Rightarrow$ $\vartheta(\tau, x) \in J^{-}_{\frac{n}{2}, m}$

(2) $F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $p(s,t) = e^{-\pi (s-t)^2}$, $X_0 = (1,1)$

$\Rightarrow$ $\vartheta(\tau, z) = A(\tau, z)$

(3) $F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $X_0 = (m_2, m_1)$ $\sum (m_1 s - m_2 t) e^{2\pi i \left(st\tau + \frac{(m_1 s - m_2 t)^2}{2m} i v + (m_1 s + m_2 t) z \right)}$

In general you do not **obtain** elements of $J^{\pm}_{k,m}$, you have to do a little bit more. What one can do is

(1) (compute the) holomorphic projection of $\vartheta(\tau, z)$

(2) **use** $\vartheta(\tau, z)$ as a kernel for lifting maps:

Set $O(F) = \{ g \in GL_n(\mathbb{R}) \mid g^t F g = F \}$, set

$R(\tau, z; g) := \vartheta(\tau, z; p^g)$ $\quad (p^g(X) := p(gX))$

then

$R(\tau, z; \gamma g) = R(\tau, z; g)$ for all $\gamma \in O(F)_{\mathbb{Z}}$;

use $R(\tau, z, g)$ as a kernel function to establish a connection

Jacobi forms $\longleftrightarrow$ automorphic forms on $O(F)$ w.r.t. $O(F)_{\mathbb{Z}}$.

$\phi \longrightarrow \langle \phi \mid R(\cdot; g) \rangle$

$\langle F \mid R(\tau, z; \cdot) \rangle \longleftarrow F$

This fits into the general philosophy: "theory of dual reductive pairs".
In the following we shall discuss the perhaps most important example of this.