generators are

$$A(\tau,z) = \sum_{s,t\in\mathbb{Z}} e^{\pi i\left(st\tau + \frac{(s-t)^2}{2} iv + (s+t)z\right)} \in J^+_{1,1},$$

$$B(\tau,z) = \frac{\partial}{\partial\bar\tau} A + \frac{\pi i}{12} A(\tau,z)\, E_2(-\bar\tau) \in J^+_{3,1}$$

$(E_2(\tau) = 1 - 24\sum_{\ell\geq1} \sigma_1(\ell) q^\ell)$. Then

$$J^+_{k,1} = M_{k-1}(1) A \oplus M_{k-3}(1) B.$$

The last example shows a general fact:

Theorem (W.) $\dim_{\mathbb{C}} J^{\pm}_{k,m} < \infty$, $J^{\pm}_{k,m} = 0$ for $k \leq 0$

proof not deep!

2. How to construct Jacobi forms (Jacobi theta series)

Aside from the Eisenstein series that Zagier constructed there is one other main source to construct Jacobi forms:

Theorem (W.) Let F be symmetric, non-singular, integral $n\times n$, even diagonal. Let $p(X)$ a function on $\mathbb{R}^n$ such that $p(X)\, e^{-\pi X^t F X}$ is a Schwartz function. Assume

$$\left(\frac{-1}{4\pi}\nabla^t F \nabla + X^t\cdot\nabla\right) p = \left(k - \frac{n}{2}\right) p \quad \text{for some } k\in\mathbb{Z}.$$

Let $X_0 \in \mathbb{Z}^n$.

Set

$$m = \frac{1}{2} X_0^t F X_0, \quad D = (-1)^{\frac{n}{2}} \det F, \quad \ell = \text{level of } F.$$

$$\vartheta(\tau,z;p) = \vartheta(\tau,z) := \sum_{X\in\mathbb{Z}^n} p\left(\sqrt{v}\left[X + \frac{y}{v} X_0\right]\right) e^{\pi i (X^t F X \tau + 2 X^t F X_0 z)}$$

Then

$$\vartheta\left(\frac{a\tau+b}{c\tau+d}, \frac{z+\lambda\tau+\mu}{c\tau+d}\right)(c\tau+d)^{-k} |c\tau+d|^{k-\frac{n}{2}} e^{2\pi i m\left(\frac{-c(z+\lambda\tau+\mu)^2}{c\tau+d} + \lambda^2\tau + 2\lambda z\right)}$$

$$= \left(\frac{D}{d}\right) \vartheta(\tau,z)$$

for all $A = \begin{pmatrix} a & b \\ c & d\end{pmatrix} \in \Gamma_0(\ell)$, all $\lambda,\mu\in\mathbb{Z}$.