① 29. Juni 89 / Vorlesungsvertretung Zagier

In his last lecture Zagier gave a very rough overview of the theory of Jacobi forms. In particular he summarized – or better to say – touched on the main connections between Jacobi forms and elliptic modular forms. Actually Jacobi forms are a very nice tool to study modular forms. There are more than one reason why this is true. In the next 75 minutes I shall try to verify this statement by explaining in more detail one aspect of this Jacobi form tool.

I shall start from the side of Jacobi forms. Our guide line will be first to construct Jacobi forms. We will see how this is possible using elliptic modular forms. But then this procedure will bounce back in a very crude way, and we will reobtain really nice results about elliptic modular forms. I shall stick throughout to the case of weight two; but all that I shall do can also be done for higher weight (but then ~~without~~ of course – with more technicalities. We start with

1. Review of Jacobi forms

What are Jacobi forms?

<u>Definition</u> Let $m \in \mathbb{Z}_{>0}$, $k \in \mathbb{Z}$. A function $\phi(\tau, z)$ $(\tau \in \mathfrak{H}, z \in \mathbb{C})$ is called a holomorphic (skew-holomorphic) Jacobi form of weight k and index m if it satisfies:

(i) $\phi(\tau, z)$ is smooth (holomorphic) in z

(ii) $\phi(\tau, z)$ is periodic in each variable with period 1; its Fourier expansion has the form

$$\phi(\tau, z) = \sum_{\substack{\Delta, r \in \mathbb{Z} \\ \Delta \equiv r^2 \bmod 4m}} c_\phi(\Delta, r)\, e^{2\pi i \left(\frac{r^2 - \Delta}{4m} u + \frac{r^2 + |\Delta|}{4m} iv + rz \right)}$$

where