Theorie zur Lösung (und Erz. von Formeln (3),(4))

$k = 2,3,4,\dots$, $\Gamma \underset{\text{endl.}}{\subseteq} \Gamma_1$

$G = GL(2,\mathbb{Q})$ op. auf $\mathbb{C}[X,Y]_{k-2}$: $A\cdot P(X,Y) := P(A^{-1}\binom{X}{Y})$

$\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]$: $A \cdot \sum n_s (s) := \sum n_s (As)$

exakte Sequ. von G-Moduln

$$0 \to V_k \xhookrightarrow{i} \mathbb{C}[X,Y]_{k-2} \otimes \mathbb{Z}[\mathbb{P}^1(\mathbb{Q})] \xrightarrow{\deg} \mathbb{C}[X,Y]_{k-2} \to 0$$

$$\sum P_s \otimes s \mapsto \sum P_s$$

$$Co_k(\Gamma) := H_0(\Gamma, V_k) \xrightarrow{i_*} H_0(\Gamma, \mathbb{C}[X,Y]_{k-2} \otimes \mathbb{Z}[\mathbb{P}^1(\mathbb{Q})])$$

$$CCo_k(\Gamma) := \ker i_* .$$

Satz Es gibt eine perfekte Paarung

$$(f,\sigma) \mapsto \int^\sigma f$$

zw. $\tilde S_k(\Gamma) = S_k(\Gamma) \oplus \overline{S_k(\Gamma)}$ und $CCo_k(\Gamma)$

($k=2$; $\sigma = (r)-(s)$: $\int^\sigma f = \int_r^s f(\tau)d\tau$)

Folg. $f \mapsto \sigma_f$, wo $\int^{\sigma_f} g = \int_{\Gamma\backslash\mathfrak{H}} [f,g]$,

def. $\tilde S_k(\Gamma) \xrightarrow{\cong} CCo_k(\Gamma)$ ($k=2$: $[f,g] = f(\tau)d\tau \wedge g(\tau)d\bar\tau$ ($f \in S_k(\Gamma)$, $g \in \overline{S_k(\Gamma)}$))

Folg

Def. 1 $g \overset{\in Co_k(\Gamma)}{,}\ \sigma \in CCo_k(\Gamma)$:

$$g \#_\Gamma \sigma = \int^g f, \quad \text{wo } \sigma \overset{f \mapsto A}{=} \sigma_f .$$

~~P mit verallg. Def.~~

Def. 2 k gerade

$Q(X,Y) = aX^2 + bXY + cY^2$ pos. def., $a,b,c \in \mathbb{Z}$, $D = b^2 - 4ac \neq \square$, definiere ~~$[C_Q]$~~

$C_Q := Q^{k/2} \otimes ((\lambda_+) - (\lambda_-)) \in V_k^{\mathbb{R}}$, $\lambda_\pm = \dfrac{-b \pm \sqrt{D}}{2a}$.

Def. $[C_Q] \in CCo_k(\Gamma)$ via

$$\int^{[C_Q]} f = \int_{\lambda_-}^{\lambda_+} Q(\tau,1)^{k/2} f(\tau)\, d\tau$$