<u>Theorem</u> Let F = Hecke eigenf. in $S_k(\Gamma_2)$, $G \in S^{Maaß}(\Gamma_2)$, then

$$D_{F,G}(s) = \langle \varphi_1 | \psi_1 \rangle Z_F(s).$$

<u>Remark</u> The proof uses only the characteristic algebraic identities of $Z_F(s)$ listed above, no analytic properties of Z_F.
This Theorem immediately yields two new problems:

1) If F = Hecke-eigenf., $G \notin$ Maaß space (e.g. $G = F$), which $D_{F,G}(s)$, ~~[illegible]~~ with the

$$D_{F,F} - \binom{\text{suitable}}{\text{const}} D_{V|\varphi_1, V\varphi_1} = \text{holomorphic in } \mathbb{C},$$
same funct. eqn as $Z_F(s)$.

Is this equal to a lin. comb. of Andrianov zeta functions, or do we get counterexamples to the general conjecture:
each D-series which can be continued to $\mathbb{C}$ with the same functional eqn. of the Andrianov-zeta functions of which is a lin.-comb. of such zeta functions.

2) F Hecke eigenf. $\Rightarrow \varphi_1 \neq 0$
(This would give via the last Theorem a new and short proof of the analytic continuation and functional eq. of the Andrianov zeta function).

Note that 1) can certainly be clarified by explicit calculations; with a system as here in Dorden like Poor's this should be no problem. Also with respect to 2) explicit calculation could help to find the correct conjecture.