<u>Theorem</u> (~ Kohnen)

Let $F, G \in M_k^{cusp}(\Gamma_2)$, $F = \sum \varphi_m e^{2\pi i m \tau'}$, $G = \sum \psi_m e^{2\pi i m \tau'}$.

Set

$$D_{F,G}(s) = \zeta(2s-2k+4) \sum_{n=1}^{\infty} \frac{\langle \varphi_n | \psi_n \rangle}{n^s}.$$

Then

- $D_{F,G}(s)$ converges for $\mathrm{Re}\, s > k+1$,
- can be meromorphically continued to $\mathbb{C}$ with a possible pole only at $s=k$ and with residue equal to $\frac{2^{2k}\pi^{k+2}}{(k-1)!} \langle F | G \rangle$

$$\left(\langle F, G \rangle = \int_{\Gamma_2 \backslash \mathfrak{H}_2} F \bar{G} \, |Y|^{k-3} \, dX \, dY \quad (Z = X + iY) \right),$$

- it satisfies

$$D^*_{F,G}(s) = (2\pi)^{-2s} \Gamma(s) \Gamma(s-k+2) D_{F,G}(s) = D^*_{F,G}(2k-2-s).$$

Note that for even k the functional equation of $D_{F,G}(s)$ is identical with the functional equation satisfied by the spinor zeta function connected to elements of $S_k(\Gamma_2)$. Of course we have to investigate what the connection between $D_{F,G}(s)$ and the Andrianov zeta function might be.

- Let $F \in S_k(\Gamma_2)$ be a Hecke eigenform, then $D_{F,F}(s)$ and $Z_F(s)$ satisfy the same functional equation.
- But note: $F \notin$ Maass then $Z_F(s)$ is holomorphic, whereas $\mathrm{res}_{s=k} D_{F,F}(s) \approx \langle F, F \rangle \neq 0$

~~Th~~

What kind of L and prove ...

*