(7)

So far I know there are only two results in the sense of this program. The first one is due to Maaß:

Theorem (Maaß, 1979)

There exist $V_m : J_{k,1} \to J_{k,m}$

$$\left(\sum c(n,r)\, e^{2\pi i(\ldots)} \mapsto \sum c^*(n,r)\, e^{2\pi i(\ldots)}, \right.$$

$$\left. c^*(n,r) = \sum_{a \mid (n,r,m)} a^{k-1}\, c\!\left(\frac{mn}{a^2}, \frac{r}{a}\right), \quad c^*(0,0) = -\frac{2k}{B_{2k}}\, c(0,0) \text{ for } m=0 \right)$$

such that

$$\phi \to \mathcal{V}\phi = \sum V_m \phi\, e^{2\pi i m \tau'}$$

defines a Hecke-equivariant embedding

$$J_{k,1} \hookrightarrow M_k(\Gamma_2).$$

(image =: "Maaß space").

Remark: One has an isomorphism $I : M_k(\Gamma_1) \oplus S_{k+2}(\Gamma_1) \xrightarrow{\cong} J_{k,1}$

$$(f,g) \mapsto fA + \frac{2}{k}\left(\frac{d}{dq} f\right) B + gB$$

$$\Big(A = q^{-6} \sum s^2 (-1)^r q^{\frac{s^2+r^2}{4}} \zeta^r,$$

$$B = \text{——— without } s^2 \text{ ———} \Big).$$

Then $\psi_k = VI(E_k)$ $k=4,6$, $\chi_{10} = VI(0,\Delta)$, $\chi_{12} = VI(\Delta,0)$.

Thus the generators for the graded ring of Siegel modular forms can be written down explicitly.

The second result is known and is due to joint work with Kohnen.