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The Maass space was discovered by numerical calculations.

L 78 Kurokuma published in Involutiones numerical examples for Hecke eigenvalues of Siegel modular forms of degree two. On the basis of his examples he and Saito conjectured the existence of a Siegel subspace in $S_k(\Gamma_2)$ which is Hecke-equivalently isomorphic to $S_{k-2}(\Gamma_2)$. Maass, Ziering proved this conjecture, this was the law of birth of the Jacobi forms.

To explain ~~these~~ ^{consider the Fourier expansion of} ~~expand~~ a Siegel modular form with respect to the variable z , this is the so called Fourier Jacobi development:

$$F = \sum_m \varphi_m e^{2\pi i m z}$$

of course $\varphi_m = \sum_{n \in \mathbb{Z}^2} a_{\Gamma}(n, \nu, m) e^{2\pi i (n\tau + \nu z)}$.

Now F has by definition a certain automorphic behaviour with respect to Γ_2 and it is clear that therefore the $\varphi_m(\tau, z)$ must also show some automorphic behaviour with respect to some subgroup of Γ_2 .

The official answer to this question is known and it runs as follows.

Let $\Gamma_1^2 = \Gamma \times \mathbb{Z}^2$. Then Γ_1^2 acts on $\mathbb{H} \times \mathbb{C}$ by

$$\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix} \lambda, \mu \right) (\tau, z) = \left(\frac{a\tau + b}{c\tau + d}, \frac{z + \lambda\tau + \mu}{c\tau + d} \right)$$

and a function $\phi(\tau, z)$ by

$$\phi \Big|_{k, m} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \phi \left(\frac{a\tau + b}{c\tau + d}, \frac{z}{c\tau + d} \right) e^{2\pi i m \left(\frac{-c z^2}{c\tau + d} \right)}$$

$$\phi \Big|_{k, \mu} (\lambda, \mu) = \phi(\tau, z + \lambda\tau + \mu) e^{2\pi i m (\lambda^2 \tau + 2\lambda z)}$$

Here k, m are integers.