③

We shall see below how one can obtain effective formulas for the $\psi_4, \psi_6, \chi_{10}, \chi_{12}$.

A Deeper ~~result~~ insight into the nature of the Fourier coefficients is provided by the so-called "Hecke theory" which – in our case of Siegel modular forms of degree two was essentially developped by Andrianov.

<u>Theorem</u> (Andrianov) There exists a sequence of (natural) Hecke operators $T(\ell)$ ($\ell=1,2,3,\dots$) on $M_k(\Gamma_2)$ such that the following holds true:

1) $M_k(\Gamma_2)$ has a basis of simultaneous eigenforms for all operators $T(\ell)$; the eigenvalues are always $\in \overline{\mathbb{Q}}$.

2) If F is a simultaneous eigenform, $F|T(\ell) = \lambda(\ell)F$, then

a) $Z_F(s) := \zeta(2s-2k+4)\sum_1^\infty \frac{\lambda(\ell)}{\ell^s}$ has an Euler product $\prod_p \frac{1}{Q_p(s)}$, where $Q_p(s)$ = polynomial of degree 4 in p^{-s}

b) For any fund. discr. $D<0$, any character $\chi : h(D) =$ classes of binary forms of discr. D $\longrightarrow S^1$:

$$\sum_{Q\in K(D)} \chi(Q) \sum_1^\infty \frac{a_F(\ell Q)}{\ell^s} = \text{cst.}\left(\sum_{Q\in K(D)} \chi(Q)\sum_1^\infty \frac{r_Q(n)}{n^s}\right)^{-1} Z_F(s)$$

($r_Q(n)$ = # of repr. of n by Q)

c) $Z_F^*(s)$ ~~can~~ converges absolutely for $\mathrm{Re}\, s \gg 0$ and can be meromorphically continued to $\mathbb{C}$ and one has

$$Z_F^*(s) := (2\pi)^{-2s}\,\Gamma(s)\,\Gamma(s-k+2)\, Z_F(s) = (-1)^k Z_F^*(2k-2-s).$$

These are rather deep facts, at least deep in the sense that their proof is very hard. Especially the proof of c) is very long but I shall show you below a very short proof – which however uses a certain ~~at least modulo a~~ conjecture.