F is periodic in each variable; thus F has a Fourier development

$$F = \sum_{n,r,m \in \mathbb{Z}} a_F(n,r,m)\, \cancel{q^n \zeta^r q'^m}\ \cancel{(q = e^{2\pi i \tau},\ q' = e^{2\pi i \tau'})}\ e^{2\pi i (n\tau + r z + m\tau')}.$$

By the "Koecher principle"

$$a(n,r,m) = 0 \text{ unless } n,m \geq 0,\ r^2 - 4mn \leq 0;$$

by ⊛ with

$$g = \left(\begin{array}{c|c} \alpha & 0 \\ \hline 0 & \alpha \end{array}\right)$$

it follows that

$$a(n,r,m) = a(n',r',m') \text{ if } \exists A \in SL_2(\mathbb{Z}) \text{ such that } [n,r,m] \circ A = [n',r',m'].$$

here $[n,r,m] = nX^2 + rXY + mY^2$, $([n,r,m] \circ A)(X,Y) = [n,r,m](A^t(X,Y))$.

thus the coefficients of F may be viewed ~~as~~ a map

~~integral positive definite quadratic forms modulo $SL_2(\mathbb{Z})$~~ $\to \mathbb{C}$

$$a_F : \mathbb{D} := \left(\begin{array}{l} SL_2(\mathbb{Z}) - \text{equiv. classes of} \\ \text{integral, semi-positive definite} \\ \text{binary quadratic forms} \end{array}\right) \longrightarrow \mathbb{C}.$$

Let us introduce at this point another notation: $S_k(\Gamma_2)$ = subspace of cusp forms in $M_k(\Gamma_2)$ = Space of $F \in M_k(\Gamma_2)$ with $a_F(Q) = 0$ unless Q is strictly positive definite.

The problem is ~~then~~

What maps $\mathbb{D} \to \mathbb{C}$ occur as Fourier coefficients of Siegel modular forms?

By a result of Igusa at least
~~At least~~ it is possible to tabulate ~~a basis for~~ the Fourier coefficients for a basis of $M_k(\Gamma_2)$ for any given k. ~~Namely one~~

~~has~~ Since we are interested in this talk only in even weight we state Igusa's result only for this case.

__Theorem (Igusa)__ Let $\varphi_4, \varphi_6, \chi_{10}, \chi_{12}$ be non-zero forms in the one dimensional spaces $M_4(\Gamma_2), M_6(\Gamma_2), S_{10}(\Gamma_2), S_{12}(\Gamma_2)$.

Then $$M_{2*}(\Gamma_2) = \bigoplus_{k \in \mathbb{Z}} M_{2k}(\Gamma_2) = \mathbb{C}[\varphi_4, \varphi_6, \chi_{10}, \chi_{12}].$$