(1) **Bordeaux, 7.12.89**

Open problem in the theory of Siegel modular forms of degree 2

The topic that I shall talk about does not really belong to algorithmic number theory. ~~Nevertheless~~ But it is a topic with many open problems and gaps – and there ~~gaps~~ realistic hope that these gaps can at least be illuminated by explicit calculations. One of the reasons for the many open questions is – to my opinion – the lack of examples.

The topic I shall speak about is

Siegel modular forms of degree two.

To start with let me recall the official definition of a Siegel modular form of degree two, ~~[illegible]~~ these are functions which satisfy:

$$(*)\begin{cases} F: \mathfrak{H}_2 \to \mathbb{C} \text{ holomorphic, where } \mathfrak{H}_2 = \{Z \in \mathbb{C}^{2\times 2} \mid Z^t = Z,\ \operatorname{Im} Z > 0\} \\ \exists k \in \mathbb{N}:\ F|_k g = F\left((AZ+B)(CZ+D)^{-1}\right)|CZ+D|^{-k} = F(Z) \quad \forall g = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma_2 \end{cases}$$

where $\Gamma_2 = Sp_2(\mathbb{Z}) = \{g \mid g\left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)g^t = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)\}$.

If you are not acquainted with this symplectic notions, there will be a more concrete description of the Siegel modular forms below.

Let $M_k(\Gamma_2)$ = space of all functions satisfying (*).

Then it is known that

$$\dim M_k(\Gamma_2) < \infty.$$

~~Any Siegel modular forms $F = F(\tau, z, \tau')$ ($Z =$~~

Write $Z \in \mathfrak{H}_2$ as $Z = \left(\begin{smallmatrix} \tau & z \\ z & \tau' \end{smallmatrix}\right)$, then $\tau, \tau' \in \mathfrak{H}$ = usual upper half plane, $z \in \mathbb{C}$ and $\Im(\tau)\Im(\tau') - \Im(z)^2 > 0$, and any Siegel modular form F can be viewed as a function $F = F(\tau, z, \tau')$.

By the transformation law (*) with $g = \left(\begin{smallmatrix} 1 & * \\ 0 & 1 \end{smallmatrix}\right)$ it is easily seen that