

Of course, here we are used that $\mathcal{D}_{k,1}$ is spanned by Poincaré-series.

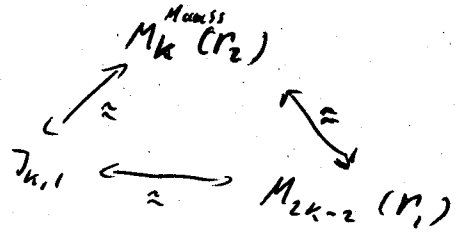
This theorem immediately yield two new problems:

- 1) If $F =$ Hecke-eigenform, $G \neq$ Maass, e.g. $G=F$ (what is $\mathcal{D}_{F,G}$ (S)). Note that $\mathcal{D}_{F,F} =$ (suitable) $\mathcal{D}_{\phi_{11V}, \phi_{11V}} =$ holomorphic on \mathbb{C} , has the same functional equation as Andriaman's zeta functions.
 - ↳ it equal to a linear combination of Andriaman-zeta functions or otherwise ~~it is counter example~~ obtain counter examples to the general conjecture of Andriaman: each Dirichlet series which can be anal. continued to \mathbb{C} and has the functional equation of the spinor zeta functions of \mathfrak{g} is linear comb. of such spinor zeta functions.

- 2) ~~↳ it is~~
 F Hecke eigenform $\stackrel{?}{\implies} \phi_1 \neq 0$
 This would give via the last theorem a new proof for the analytic continuation and no faith of the spinor zeta function.

Finally I would like to mention another nice

Corollary. As is showed above we have Hecke-eigenform isomorphisms:



Let F be a Hecke-eigenform in $M_k^{Maass}(\rho_2)$ and f the corresponding elliptic modular form. ~~The~~ ^{vs. Jacobi}

$$L(\phi, s) \zeta(s-k+1) \zeta(s-k+2) = \mathbb{Z}_F(s) = \mathcal{D}_{F,F}(s) \langle \phi/\phi \rangle^{-1}$$

Comparing residues at $s=k$ on both sides we get

Corollary
$$L(\phi, k) = \frac{C \cdot 4^k \pi^{-k}}{(k-1)!} \frac{\langle F, F \rangle}{\langle \phi/\phi \rangle} \quad (F = \phi/V)$$