Of course, here we also used that $J_{k,1}$ is spanned by Poincaré-series.

This Theorem immediately yield two new problems:

1) If $F$ = Hecke-eigenform, $G \notin$ Maass, e.g. $G=F$, what is $D_{F,G}(s)$. Note that $D_{F,F} - \binom{\text{suitable}}{\text{constant}} D_{\varphi_1|V,\varphi_1|V}$ = holomorphic on $\mathbb{C}$, has the same functional equation as Andrianov's zeta functions.
Is it equal to a linear combination of Andrianov-zeta functions or do we ~~it a counterexample~~ obtain counterexamples to the general conjecture of Andrianov: each Dirichletseries which can be hol. continued to $\mathbb{C}$ and has the functional equation of the spinor zeta functions of wt $k$ is linear comb. of such spinor zeta functions.

2) ~~Is it true~~
$F$ Hecke eigenform $\overset{?}{\Longrightarrow}$ $\varphi_1 \neq 0$
This would give via ~~the~~ last Theorem a new proof for the analytic continuation and no poles of the spinor zeta function.

Finally I would like to mention another nice
Corollary. As indicated above we have Hecke-equivariant isomorphisms:

$$\begin{tikzcd} & M_k^{Maass}(\Gamma_2) \arrow[dr, leftrightarrow, "\cong"] & \\ J_{k,1} \arrow[ur, leftrightarrow, "\cong"] \arrow[rr, leftrightarrow, "\cong"'] & & M_{2k-2}(\Gamma_1) \end{tikzcd}$$

Let $F$ be a Hecke-eigenform $\in M_k^{Maass}(\Gamma_2)$ and $f, \varphi$ the corresponding elliptic modular form resp. Jac. fm. The

$$L(f,s)\,\zeta(s-k+1)\,\zeta(s-k+2) = Z_F(s) = D_{F,F}(s)\,\langle\varphi|\varphi\rangle^{-1}$$

Comparing residues at $s=k$ on both sides we get

Corollary
$$L(f,k) = \frac{6\cdot 4\pi^k}{(k-1)!}\,\frac{\langle F,F\rangle}{\langle\phi|\phi\rangle} \qquad (F=\phi|V).$$