But before let me say a few words about the proof:
The idea is to mimic the proof of the elliptic modular form case, i.e. the Rankin Selberg method.
For this we have to find an Eisenstein series of Klingen-Siegel-type

$$E_s(Z) = \sum_{g \in \mathcal{C} \backslash \Gamma_2} \gamma_s(g\langle Z\rangle) \quad \text{with suitable } \gamma_s(Z).$$

Then we have by the usual trick of unfolding the integral

$$\int_{\Gamma_2 \backslash \mathfrak{H}_2} F \overline{G} \sum_{g \in \mathcal{C} \backslash \Gamma_2} \gamma_s(g\langle Z\rangle) \, |Y|^{k-3} \, dX\, dY$$

$$= \int_{\mathcal{C} \backslash \mathfrak{H}_2} F \overline{G} \, \gamma_s(Z) \, |Y|^{k-3} \, dX\, dY \; ;$$

Now
$\mathcal{C} \backslash \mathfrak{H}_2 = \{ (\tau, z, \tau') \mid (\tau, z) \in \Gamma_1^J \backslash \mathfrak{H} \times \mathbb{C}, \; 0 \le \operatorname{Im} \tau' \le 1 \}$, then we can continue

$$= \int_{\Gamma_1^J \backslash \mathfrak{H} \times \mathbb{C}} \int_{0 \le u' \le 1} \int_{v' > \frac{y^2}{v}} \sum_{m,n} \varphi_m \overline{\psi_n} \, e(m\tau' - n\overline{\tau}') \, \gamma_s(Z) \, |Y|^{k-3} \, du\, dv\, dx\, dy\, du'\, dv'$$

and now for $\gamma_s(Z) = \frac{|Y|^s}{v^s}$

$$= \int_{\Gamma_1^J \backslash \mathfrak{H} \times \mathbb{C}} \sum_m \varphi_m \overline{\psi_m} \left(\int_{v' > \frac{y^2}{v}} e^{-4\pi m v'} \, \frac{\cancel{\gamma_s(Z)}\, (v v' - y^2)^{k-3+s}}{v^s} \, dv' \right) du\, dv\, dx\, dy$$

and finally setting $v' + \frac{y^2}{v} \to v'$ we get

$$= \sum_m \langle \varphi_m | \psi_m \rangle \int_0^\infty e^{-4\pi m v'} \, v'^{\,k-2+s} \, \frac{dv'}{v'} \, ,$$

thus

$$D_{F,G}(k-2+s) = \langle F, E_s G \rangle \, (4\pi)^{k-2+s} \, \Gamma(k-2+s)^{-1} \, \zeta(2s).$$