(3a)

Lemma (*) $\langle \varphi_m | \psi_m \rangle = O(m^{k})$

proof

$$\varphi_m = \int_{iC}^{iC+1} F(\tau)\, e^{-2\pi i m \tau'}\, d\tau' \qquad \text{for any } C > y^2/v\,,$$

thus

$$|\varphi_m| < * \, (vC - y^2)^{-k/2}\, e^{2\pi m C} \qquad \text{since } F(\tau)\, |Y|^{k/2} \text{ is bounded on } \mathfrak{h}_2 .$$

Choose $C = \frac{y^2}{v} + \frac{1}{m}$ to obtain

$$|\varphi_m| < * \; m^{k/2}\, e^{2\pi m \frac{y^2}{v}}\,;$$

and with a similar estimate for ψ_m we obtain the assumption.

$\Leftarrow$