

A certain Dirichlet series associated to Siegel modular forms of degree 2.

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Before I start to explain in more detail results I should perhaps make some general remarks concerning the topic of this talk. I shall speak about the connection between Siegel modular forms of degree two and Jacobi forms. More precisely, every Siegel modular form of degree two has a so-called Fourier-Jacobi expansion, i.e. a Fourier expansion so that the Fourier coefficients are again automorphic forms: namely Jacobi forms. Now, the theory of Jacobi forms is ^{meanwhile} very well developed. One understands Jacobi forms much better than Siegel modular forms of degree two. So it seems to be a good idea to start or to reconsider investigations in the direction of the following program:

Study Siegel modular forms of degree two via their Fourier-Jacobi-expansion.

The results that I would like to explain here can be viewed as part of such a program. Finally I should mention that these results come from joint work with W. Kohnen.

~~Before I can start and explain the results I~~

So let me begin by introducing or reminding you of some basic notions. Fourier-Jacobi-development of a Siegel mod. form of degree 2:

Let $F \in M_k(\Gamma_2)$ ($\Gamma_2 = Sp_2(\mathbb{Z})$), then $F = F(\tau, z, \tau')$ where $\tau, \tau' \in \mathbb{H}$, $z \in \mathbb{C}$ and that $\text{Im}(\frac{\tau \bar{\tau}'}{z \bar{z}}) > 0$. Then F is especially ^{in each variable} periodic, especially it is periodic in the third variable; then it has a Fourier development with respect to the third variable:

$$F = \sum_{m \geq 0} \varphi_m(\tau, z) e^{2\pi i m \tau'}$$

If $F = \sum_{\substack{c_1, c_2, c_3 \in \mathbb{Z} \\ c_1^2 + 4c_2 c_3 < 0}} c(c_1, c_2, c_3) e^{2\pi i(c_1 \tau + c_2 z + c_3 \tau')}$ then $\varphi_m(\tau, z) = \sum_{\substack{c_1, c_2 \in \mathbb{Z} \\ c_1^2 + 4c_2 z < 0}} c(c_1, c_2) e^{2\pi i(c_1 \tau + c_2 z)}$
Now F has by definition a certain automorphic behavior with respect to Γ_2 and it is clear that this implies ~~an~~ ^{an} automorphic behavior therefore the $\varphi_m(\tau, z)$ must also show some automorphic behavior with respect to ~~some~~ ^{some} subgroup of Γ_2 .