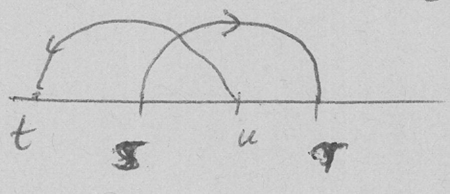
3/ A third kind of re-markable formula shows up if you consider intersection numbers:

$$((r)-(s))\circ((t)-(u))$$

$:=$ #-numbers $= +1$ (in this picture)



you can extend to $\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0$: by defining

$$\sum_r c_r(r)\circ\sum_s d_s(s)$$

$$:= \frac{1}{2}\sum_{\underline{\underline{r,s\in\mathbb{Q}}}} c_r d_s \operatorname{sign}(s-r)$$

This is $GL(2,\mathbb{Q})^+$-invt: $AD\cdot AD' = D\cdot D'$

Now fix $D \in \mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0$:

then

$$\lambda_E(D) := \sum_{D'\in\Gamma_0(N)D}(D')\circ E$$

actually infinite sum, cusps are causing trouble

but

$$\sum_{\substack{D'\in\Gamma_0(N)D\\ D'\cap E}} D'\cdot E \;+\; \sum_{D'} D'\cdot E$$

finite sum

D' has a cusp with E in common (e.g. $\infty\in E$, then $(\infty)-(0) = T^i((0)-(\infty))$)

here you can "renormalize"

gathered formula are very clumsy (never really worked them out). (but if you assume that E is cuspidal, you can throw away the second sum)

Howmuchever it still remains the mystery of the Heegners. Of course you can calculate the ϕ but this for that never told me anything. What you can do, is of course looking at the Eisenstein series!