in the rest of the talk we are mainly interested in EI.
This exist in diff. formulations.
We give here one that I like very much:
It is completely elementary and simple (you do not even have to know mod. forms to understand), good for second year students.
Here $M \supseteq \mathbb{T}$ directly derived from ~~group~~ $\Gamma_0(N)$.
(not new – I expl. this to several people in conf. also in a decade ago, but I am always coming back to it – many open questions).

$$\begin{cases} M := \mathrm{Hom}_{\Gamma_0(N)}\left(\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0,\ \mathbb{C}[X]_{k-2}/\mathbb{C}\right) \\ (T(n)\lambda)(\mathcal{D}) = \sum_{M \in \Gamma_0(N)\backslash \mathcal{M}(n)} \tilde{M}\cdot\lambda(M\mathcal{D}) \end{cases}$$

for $k=2$ this even simplifies ✗

(here $\mathbb{Z}[\mathbb{P}^1(\mathbb{Q})]^0 = \mathcal{D}$ "divisors":
nat. action of $Gl(2,\mathbb{Q})$ on $\mathcal{D}$)
$\mathcal{M}(n) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{Z}^{2\times 2} \mid N \mid c,\ (a,N)=1,\ \det = n \right\}$

~~though unnecessary for computations~~
not exactly $M_k(N) \underset{\mathbb{T}}{\cong} M$ but
$$M_2(N) = \left\{ \sum_{n\geq 1} \lambda(T(n)\mathcal{D})\, q^n : \mathcal{D} \in \mathcal{D} \right\}$$ "up to constant terms" (i.e. $=$ after. $\cap\, q\mathbb{C}[[q]]$)

before we discuss this setting w.r.t. to expl. computations or theoretic significance, we indicate ~~why this is a $\mathbb{T}$-equivalence~~
how this is related to mod forms. – though this expl. relation not important for discussion.