Quelques remarques sur le calcul des formes modulaires

modular forms : $M_k(N)$ wrt $\Gamma_0(N)$

sub spaces $S_k(N)$ cusp forms

$S_k(N)^{new}$ etc.

k even int. ≥ 2, ever mostly $k=2$

$\mathbb{T} = \mathbb{T}_N$ algebra generated (over $\mathbb{Q}$) by all $T(n)$ $n=1,2,\dots$

2 essentially diff.

√ methods to calculating $M_k(N)$, in. partic. HEF's

1/ trace formula !

~~a/ theta series~~

~~b/ modular symbols~~

~~fact~~ $S_k(N)^{new}$ $\mathbb{T}$-mod. rank 1

generator $\sum_{n\geq 1} \underbrace{\mathrm{tr}(T(n), S_k^{new}(N))}_{\text{exp. known (cumb. but elem. formulas)}} q^n$

for a/ theta series ($\mathbb{T}$-adic given by Brandt-matrices)

for b/ (mod p) $\mathbb{T}$-modules $H^1_{cusp.}(\Gamma_0(N), \mathbb{C}[X]_{k-2})$ ℓ-adic cohom.

(Hecke, Wang, ...)

or - say $k=2$ - $\mathbb{T}$-module $H_1(X_0(N), \mathbb{Q})$ (ess.)

(Manin, ~, Cremona, Merel, Mülla (Frey), Stein, Quer, ...)

b/ Mestre :