(9)

On trouve

$$\Theta_E(t,\mathfrak{a}) = \mu(E_{\mathbb{R}}/E) + \sum_{\substack{\alpha\in\mathfrak{a}/E \\ \alpha\neq 0}} \sum_{\cancel{\varepsilon\in E}} \int_{E_{\mathbb{R}}/E} \underbrace{\exp(-c_{\alpha} \textstyle\sum_v e_v |\alpha|_v^2 (|\varepsilon|_v y_v)^2 t)}_{} \; d\mu(x)$$

$|x\alpha|^2$

$$= \quad - \quad + \sum_{\alpha} |E_{tor}| \int_{E_{\mathbb{R}}}$$

$$= \mu(E_{\mathbb{R}}/E) + |E_{tor}| \sum_{\substack{\alpha\in\mathfrak{a}/E \\ \alpha\neq 0}} \underbrace{\int_{E_{\mathbb{R}}} \exp(-c_{\mathfrak{a}} t\, |x\alpha|^2)\, d\mu(x)}_{f_\alpha(t)}$$

et on a

$$\Theta_E(t,\mathfrak{a})\, t^{[L:\mathbb{Q}]/2} = \Theta_E\left(\tfrac{1}{t}, (\mathfrak{a}\mathfrak{d})^{-1}\right)$$

$\left(\text{d'après } \vartheta(|y_v^{-1}|, \mathfrak{a}) \prod y_v^{e_v/2} = \vartheta(|y_v^{-1}|, (\mathfrak{a}\mathfrak{d})^{-1})\right)$

Donc

$$\Theta_E(t,\mathfrak{a})\, t^{[L:\mathbb{Q}]/2} \uparrow \qquad (t\to 0)$$

Dérivée ≥ 0 !

$$\mu(E_{\mathbb{R}}/E) \geq \frac{2|E_{tor}|}{[L:\mathbb{Q}]} \sum_{\substack{\alpha\in\mathcal{O}/E \\ \alpha\neq 0}} \left(\frac{-2t}{[L:\mathbb{Q}]} f_\alpha'(t) - f_\alpha(t)\right)$$

Si $E = \mathcal{O}_{L/k}^*$ et si on ~~normalise~~

$\frac{\mu(E_{\mathbb{R}}/E)}{|E_{tor}|} = \mathrm{Reg}(L/k)$ on trouve

$$\int_{E_{\mathbb{R}}} \exp(-t\,|x\alpha|^2)\, d\mu(x) = \prod_{w\in A_k} g_{p_w, q_w}\left(|N_{L/k}(\alpha)|_w^{e_w}\, t^{\frac{e_w[L:k]}{2}}\right)$$

où

$$g_{p,q} = f_{\mathbb{R}}^{*p} * f_{\mathbb{C}}^{*q} \qquad (\text{convolution de Mellin})$$

$$f_{\mathbb{R}} = \exp(-t^2) \qquad f_{\mathbb{C}} = \exp(-2t)$$

$\int \exp(-t^2)\, t^s \frac{dt}{t} \sim \Gamma(\tfrac{s}{2}) \qquad M \sim \Gamma(s)$