Table of noncongruent fundamental discriminants $D \equiv 1 \bmod 8$ and $< 10\,000$

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Let $L(E, s)$ denote the L-series of the elliptic curve $y^2 = x^3 - x$. Then $L(E, s)$ coincides with the L-series of the unique normalized cusp form of weight 2 on $\Gamma_0(32)$. One has the formula

$$L(f, D, 1) = \frac{c}{\sqrt{D}} |c(D, r)|^2,$$

for any positive fundamental discriminant D which is a square modulo 8 and for any solution r of $r^2 \equiv D \bmod 128$. Here $c(D, r)$ is the D, r-th Fourier coefficient of the (up to scalar multiplication) unique Jacobi cusp form of weight 2 and index 32 attached to f via the Shimura liftings, and c is a constant not depending on D and r. These numbers can be computed by a simple finite formula ([**S**]).

The following table lists these numbers for $D < 10\,000$. The discriminants are ordered according to the values of $c(D, r_D)/\sigma_0(D)$ whose square conjecturally gives the value of the Tate-Shafarevitch group of the elliptic curve $y^2 = x^3 - D^2 x$. (Here r_D is that solution of the equation $r_D^2 \equiv D \bmod 128$ which satisfies $0 < r_D < 32$.) If D is a congruent number, i.e. the area of a right triangle with rational sides, then $c(D, r_D) = 0$. The converse statement is conjecturally true as well (cf. [**T**]).

THE TABLE

−12 [9433, 9697]
−10 [3449, 4057, 5281, 5521, 6361, 7753]
−9 [7513, 7617, 9753]
−8 [1993, 2297, 2657, 2713, 3217, 3929, 4073, 4289, 4729, 6449, 6473, 7393, 7433, 8081, 8089, 8297, 8369, 9833]
−7 [8889]
−6 [673, 1361, 1433, 1481, 1697, 1873, 2081, 2393, 2473, 3169, 3257, 3329, 3617, 3833, 4105, 4297, 5113, 6913, 7537, 7937, 8137, 8497, 8713, 8753, 9233, 9577, 9593]
−5 [1257, 1273, 1577, 1713, 3057, 4377, 4593, 6017]
−4 [409, 569, 577, 857, 1601, 1657, 1673, 1889, 2137, 2153, 2513, 2521, 2545, 2617, 2633, 2705, 2729, 2921, 2977, 3121, 3137, 3233, 3361, 3385, 3769, 4177, 4265, 4273, 4537, 4577, 4817, 5089, 5201,

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