

(1) A large part of Hecke's work deals with this "Grand problem". Moreover, the most powerful tools to attack this problem, can all be found in Hecke's papers.

To fix thought and to become more concrete I have to review some of these tools and results.

To simplify notation we consider from now on only the case

(1a) By the way, it is exactly these Eisenstein series which Hecke considers in the cited paper. In particular he develops in this paper a method which became very famous within the last 65 years under the name "Hecke tricks".

(2) Also for the periods there are some algebraicity statements. The Fourier coefficients and the periods have undoubtedly a very deep arithmetical significance which is not completely understood today. As an illustration I give the following

(3) A more modern tool for the study of elliptic modular forms are the Jacobi forms. Indeed, Jacobi forms represent in a sense the link between Fourier coefficients and periods of modular forms.

(4) In fact, Jacobi forms are older than elliptic modular forms. The prototypes for Jacobi forms are

$$\text{prototypes: } \sum_{n \in \mathbb{Z}} q^{n^2} \cos(4\pi n z) \in J_{1, \frac{1}{2}}^{-1}(\Gamma_0(4))$$

However it was not before 10 years ago that the arithmetical significance of Jacobi forms became clear. The starting point was the monography

Th. of Jac. forms (Fuchs/Zagier)