(2)

$\mathcal{H} := \mathbb{Z}[T(\ell) \,:\, (\ell,m)=1]$ kommutativ

$M_k(\Gamma)$ halbeinf. $\mathcal{H}$-Modul

$M_k(\Gamma) = M_k^{Eis}(\Gamma) \oplus S_k(\Gamma)$ (als $\mathcal{H}$-Modul)

$S_k(\Gamma) = \bigoplus V$ kanonische Zerlegung

$S_k^{neu}(\Gamma) = \bigoplus_{\dim V=1} V$, $S_k^{alt}(\Gamma) = \bigoplus_{\dim V>1} V$

$M_k^{Eis}(\Gamma)$: Prototyp $E_4 = 1 + 240 \sum_{\ell=1}^{\infty} \sum_{d \mid n} d^3 q^\ell$ $(\in M_4^{Eis}(Sl(2,\mathbb{Z})))$ 1α

$S_k^{alt}(\Gamma)$: trivial aus $S_k(\Gamma_0(m'))$, $m' < m$

$S_k^{neu}(\Gamma) \ni f$ HEF, $L(f,s) = a_f(1) \prod_p \frac{1}{1-\lambda_p p^{-s} + \delta(p \nmid m) p^{k-1-2s}}$,

$\exists\, K = K_f / \mathbb{Q}$ endl. Körpererw. $\forall p\ \lambda_p \in K_f$ 2

Für HEF Perioden interessant
$S_2(\Gamma_0(32)) = \mathbb{C}\cdot f$, ~~$f = q + O(q^{??})$~~
Satz ("Tunnell")

D Kongruenzzahl $\Longrightarrow L(f \otimes D, 1) = 0$ ← 3

Jacobiformen

$J_{k,m}^{\pm} = \Big\{ \phi = \phi(\tau,z) : \mathfrak{H} \times \mathbb{C} \xrightarrow[\text{hol. in } z]{\text{glatt}} \mathbb{C} \ \Big|$
1) $\phi(\tau+1,z) = \phi(\tau,z+1) = \phi(\tau,z)$
2) $\phi(\frac{-1}{\tau}, \frac{z}{\tau}) = \cancel{\ldots}\, e^{2\pi i m z^2/\tau} \begin{cases} \tau^{k}\ \ldots \\ \bar\tau^{k-1} |\tau| \ \ldots \end{cases} \phi(\tau,z)$
3) $\phi = \sum_{\substack{D,r \in \mathbb{Z} \\ D \equiv r^2 \bmod 4m \\ D > 0\ \text{"+"} \\ < 0\ \text{"−"}}} c_\phi(D,r)\, e^{2\pi i \left(\frac{r^2-D}{4m} u + \frac{r^2+|D|}{4m} iv + rz \right)}$ $(\tau = u+iv)$
wo $c_\phi(D,r) = c_\phi(D, r+2m)$ $\Big\}$

$, m = 1,2,\ldots,$

~~Prototyp $\phi(\tau,z)$ (aber: meromorph, $(k,m)=(2,0)$)~~ ← 4

$\left(S_{k,m}^{\pm} : \text{" } c_\phi(0,r) = 0 \ \forall \tfrac{r^2}{4m} \equiv 0 \text{ "} \right)$

$\dim J_{k,m}^{\pm} < \infty$

$J_{k,m}^{\pm} = \mathcal{E}_{k,m}^{\pm} \oplus S_{k,m}^{\pm}$

$\phi^{\pm}$ "