$$\chi_\Delta(a) = \begin{cases} \left(\frac{\Delta/f^2}{a/f^2}\right) & \text{if } (a,\Delta) = f^2 \text{ and } \frac{\Delta}{f^2} = \text{discriminant} \\ 0 & \text{otherwise} \end{cases}$$

Thm $T(\ell)$ maps $J_{k,m}^{(+)}$ onto itself and $T(\ell)\circ T(\ell') = \sum_d d^{2k-3}\, T\left(\frac{\ell\ell'}{d^2}\right)$.
Note that the last relation is the usual relation satisfied by the usual Hecke operators on elliptic modular forms of even weight.

Proof Check $\phi|T(\ell) = \ell^{k-4} \sum_{x\in\mathbb{Z}^2/\ell\mathbb{Z}^2} \sum_{\substack{M\in SL_2(\mathbb{Z})\backslash\mathbb{Z}^{2\times 2}\\ \det M = \ell^2,\ gcd(M)=\square}} \phi^{(m)}\big|\left(\tfrac{1}{\ell}M\right)[x]$.

2. Main Theorem

Essentially the Main Theorem is

Theorem $J_{k,m}^{(+),new} \approx M_{2k-2}^{new,(+)}$ as Hecke-modules.

However instead of explaining the notion of "Jacobi-new-form", one can do the following: It is possible to glue together all such above isomorphisms to give a Hecke-equivariant map ~~from~~ $J_{k,m}^{(+)} \approx$ ~~$\mathfrak{M}^{(+)}_{2k-2}$, where $\mathfrak{M}_{2k-2}$~~ onto a "canonical subspace of $M_{2k-2}(\Gamma_0(m))$.
More precisely let

$$\mathfrak{M}_k(m) = \mathrm{Span}_{\mathbb{C}}\left\{ f\in M_k(\Gamma_0(m)) \;\middle|\; \begin{array}{l} \exists\, m'|m,\ g\in M_{2k-2}^{new}(m') \text{ such that } L(f,s) = \Big(\prod_{p|\frac{m}{m'}} Q_p(s)\Big) L(g,s) \text{ with} \\ Q_p(s) = \text{polyn. in } p^{-s} \text{ such that} \\ Q_p(s) = p^{t(\frac{k}{2}-s)} Q_p(k-s) \text{ for all } p^t \| \frac{m}{m'} \end{array} \right\}.$$

Set $\mathfrak{M}_k^{\pm}(m) = \left\{ f\in\mathfrak{M}_k(m) \;\middle|\; f|W_m := f\left(\frac{-1}{N\tau}\right)(\sqrt{N}\tau)^{-k} = \pm(-1)^{k/2} f \right\}$.