Also it is possible and easy to prove

<u>Proposition</u> Let $J^*_{*,m} = \bigoplus_k J_{k,m}$, $M_* = \bigoplus_k M_k(Sl_2(\mathbb{Z}))$. Then $J^*_{*,m}$ is a <u>free</u> M_*-module via $(F,\phi) \mapsto F(-\bar\tau)\cdot\phi(\tau,z)$.

Because of the dimension formula it is then clear that

$$J^*_{*,1} = A\,M_* + B\,M_*, \quad \text{i.e.}\quad J^*_{k,1} = A\,M_{k-1} \oplus B\,M_{k-3}.$$

<u>3rd case: arbitrary signature (a,b)</u>

Let $G = \{g \in SO(V,\cdot) \mid g z_0 = z_0\}$, ~~then~~ $G \cong SO(a-1,b)$, define

$$\vartheta(\tau,z;g) = \sum_{Z\in\Lambda} v^{\frac{a}{2}-\frac{k}{2}}\, P(\sqrt{v}\,[g^{-1}Z + \tfrac{y}{v}z_0])\; e\!\left(\tau\frac{Z^2}{2} + z\,Z\cdot z_0\right) \quad \left((\tau,z)\in\mathfrak{H}\times\mathbb{C},\ g\in G\right)$$

Then $\vartheta(\tau,z,g)$ behaves like a Jacobi form in (τ,z) and like a function on $G/\mathrm{Stab}(P)$ in g. This shows the possibility of setting up a theory of theta correspondences à la ~~Howe~~, Kudla et al for Jacobi forms. Especially Jacobi forms should be connected to elliptic modular forms ($G \cong SO(2,1) \cong SO(1,2)$). I shall not pursue this here but instead formulate the perhaps simplest, but also most important case in a more concrete language.

<u>Connection between Jacobi forms and elliptic modular forms (of even weight)</u>

~~Essentially the Main Theorem~~

<u>1.) Hecke-Theory for Jacobi forms.</u>

Here one has the following Theorem:

<u>Theorem</u> Let $\phi \in J^{(*)}_{k,m}$, $\phi = \sum C(\Delta,r)\, e\!\left(\frac{r^2-\Delta}{4m}\tau + \left\{\frac{\Delta}{4m}\,\tau-\bar\tau\right\} + rz\right)$.
For $l \geq 1$, $(l,m)=1$ set

$$\phi|T(l) = \sum_{\Delta,r}\Big(\sum_a a^{k-2}\chi_\Delta(a)\, C\!\left(\frac{l^2}{a^2}\Delta, r'\right)\Big)\, e(\ldots\Delta,r\ldots),$$

where we have: the sum over a is over all $a \mid l^2$ s.t. $\frac{l^2\Delta}{a^2}$ is a discriminant, r' is (mod $2m$) uniquely determined by: $lr \equiv ar' \ (a,l)$, $r'^2 \equiv \frac{l^2}{a^2}\Delta\ (4m)$.