As above one can prove here:

Lemma Any $\phi \in J^*_{k,m}$ ($:= J^*_{k,m}(SL_2(\mathbb{Z}))$) has a Fourier expansion of the form

$$\phi = \sum_{\substack{\Delta, r \in \mathbb{Z},\ \Delta \geq 0 \\ \Delta \equiv r^2 \bmod 4m}} C_\phi(\Delta, r)\, e\left(\frac{r^2-\Delta}{4m}\tau + \frac{\Delta}{2m} i v + r z\right),$$

where $C_\phi(\Delta, r)$ depends only on r modulo $2m$.

Also one gets from this:

Proposition Any $\phi \in J^*_{k,m}$ can be written as $\phi = \sum h_\rho(\tau)\vartheta_{m,\rho}(\tau, z)$, where $h_\rho(-\bar\tau) \in M_{k-1/2}(\Gamma(4m))$. This gives an isomorphism

$$J^*_{k,m} \approx \left(M^*_{k-1/2}(\Gamma(4m)) \otimes \mathrm{Span}_{\mathbb{C}}\{\vartheta_{m,\rho} / \rho = 1, \dots, 2m\}\right)^{SL_2(\mathbb{Z})}.$$

($h \in M^*_{k-1/2}(\Gamma(4m)) \iff h(-\bar\tau) \in M_{k-1/2}(\Gamma(4m))$)

Corollary $\dim J^*_{k,m} < \infty$ ($= \frac{km}{6} + O(1)$ (later)), (in principle) explicit trace formula for double coset operators.

Before continuing towards the Main Theorem let me write down the simplest examples for skew-holomorphic Jacobi forms:

$$A = \sum_{x,y \in \mathbb{Z}} e^{-2\pi v \frac{(x-y)^2}{2}} e^{\pi i \tau(2xy + 2z(x+y))} \in J^*_{1,1}(SL_2(\mathbb{Z}))$$

$$B = \frac{\partial}{\partial \bar\tau} A + \frac{\pi i}{12} A\, E_2(-\bar\tau) \in J^*_{3,1}(SL_2(\mathbb{Z}))$$

For B one needs a

Lemma

$$\frac{\partial}{\partial \bar\tau} + \frac{\pi i (k-\frac{1}{2})}{6} E_2(-\bar\tau)$$ changes $J^*_{k,m}$ to $J^*_{k+2,m}$.