satisfies

$$\operatorname{div}\tilde f(\tau,\cdot) = \operatorname{div} f(\tau,\cdot), \qquad \tilde f(z) = \frac{1}{z^N} + O\left(\frac{1}{z^{N-1}}\right).$$

Thus $\tilde f = f$ which means that

f transforms like a Jacobi form of weight N, index 0.

Actually

f is a meromorphic Jacobi form of weight N, index 0,

i.e. is quotient of two (hol.) Jacobi forms of suitable weights and (same) index.

Namely

$$f = \frac{\sigma(\tau,z)^N}{\sigma(\tau, z-\frac{1}{N})^N} \quad \text{with} \quad \sigma(\tau,z) = z\prod_{\substack{\gamma\neq 0\\ \gamma\in\mathbb{Z}\tau+\mathbb{Z}}}\left(1-\frac{z}{\gamma}\right)e^{\frac{z}{\gamma}+\frac{1}{2}\left(\frac{z}{\gamma}\right)^2}$$

= Weierstraß σ-function,

and

$$\varphi(\tau,z) := \eta^3(\tau)\exp\left(-\frac{\pi^2}{6}E_2(\tau)z^2\right)\sigma(\tau,z) \qquad \left(\begin{array}{l}\eta = q^{1/2}\prod_1^\infty(1-q^n)\\ E_2 = 1-24\sum_1^\infty\sigma_1(n)q^n\end{array}\right)$$

is a hol. Jacobi modular form of weight $1/2$, index $\frac{1}{2}$. The latter can be checked by doing the above ~~with the~~ more generally with theta functions instead of elliptic functions. The function φ is essentially equal to one of the standard theta functions

$$\vartheta_{m,\varrho}(\tau,z) = \sum_{\substack{r\in\mathbb{Z}\\ r\equiv\varrho\,(2m)}} q^{r^2/4m}\zeta^r \qquad (m\in\mathbb{Z}_{>0},\ \varrho = 1,2,3,\dots,2m).$$

This can be seen by applying ~~Theorem~~ the triple product identity to $\vartheta_{1,0}$ to show that $\operatorname{div}\vartheta_{1,0}(\tau,\cdot) = \left(\frac{1+\tau}{2}\right)$, then $\vartheta_{1,0}(\tau,z-\frac{1+\tau}{2}) =$ trivial theta function $\times\ \varphi(\tau,z)$ etc., ~~actually~~ $\vartheta_{1,1}$. These function are really important. Namely, that $\phi\in J_{k,m}$ has a Fourier expansion of the form

$$\phi = \sum C(\Delta,r)\, q^{\frac{r^2-\Delta}{4m}}\zeta^r, \quad C(\Delta,r) \text{ depend only on } r \bmod 2m$$

implies that

$$(*) \qquad \phi = \sum_{\varrho=1}^{2m} h_\varrho(\tau)\,\vartheta_{m,\varrho}(\tau,z)$$

and it is easy to deduce from this: