wt of $f = \frac{a_1}{2} + \frac{p}{2}$ char of $f = \varepsilon^{s a_1 + p}$

$$\text{index of } f = \frac{1}{2}\sum_i \underbrace{\sum_{t \mid a_i} \mu(a_i/t)\, t^2}_{\geq \frac{6}{\pi^2} a_i^2} \geq \frac{3}{\pi^2}\sum_{i=1}^{N} a_i^2$$

! (For S) ~~Corollary~~ For given k, m, char. one can algorithmically enumerate ~~For fixed index there is~~ all thetablocks in $J_{k,m}$ (char.)

Note δ: # thetablocks with fixed index mod powers of η = finite

• for $k \gg 0$ $J_{k,m}(\varepsilon^a)$ contains ~~al.~~ few thetablocks ($\dim J_{k,m}(\varepsilon^a) \sim$ a const $\cdot k$)

• challenge: construct thetablocks of small wt (i.e. small p in (S))

~~→~~ Stiller for wt 2 — see 86

Conclusions (experimentally verified)

• $J_{2,m}$ (char) always spanned by thetablocks (for index ≤ 150)

| However Armand Brumer found two Jacobi forms in wt 2 whose ... not span $J_{2,m}$ — index: 191?

• $J_{1,m}(\varepsilon^a)$ contains no thetablocks for $a = 0, 12, 16, 18, 20, 22$ (S)

Conjecture • $J_{k,m}(\varepsilon^a)$ always spanned by thetablocks for small k (say $k \leq 2$) possibly $k \leq 4$

• $J_{1,m}(\varepsilon^a) = 0$ for $a = 0, 12, 16, 18, 20, 22$ ~~in list~~ (S)

Thm (S) ~~dim~~ $J_{1,m}(1) = 0$

Recall $J_{1/2,m}(\varepsilon^a)$ also spanned by thetablocks.

Via $J_{1,m}(\varepsilon^a) \to M_{1/2}$ with $Mp(1,\mathbb{Z})$-module structure:

Thm. $\dim J_{1,m}(\varepsilon^a) = 0$ for $a = 0, 12, 16, 18, 20, 22$, all $m \in \frac{1}{2}\mathbb{Z}$

Thm. For integral m, $J_{1,m}(\varepsilon^8) = \langle \Theta_{a,b} \mid m = a^2 + ab + b^2,\ a, b > 0 \rangle$