Conclusion

Prop The theta block f is holomorphic at ∞

iff

$$\sum_i B(a_i x) - \sum_j B(b_j x) + \frac{n}{24} \geq 0 \quad \text{for all } x.$$

Quite interesting condition. Example first

Thm For $a, b > 0$,

$$\Theta_{a,b} := \frac{\vartheta_a \vartheta_b \vartheta_{a+b}}{\eta} \in J_{1,\, a^2+ab+b^2} \quad (\varepsilon^8)$$

proof Need to show

$$B(ax) + B(bx) + B((a+b)x) \geq + \frac{1}{24}$$

Set $ax \leftarrow x$ $\quad bx \leftarrow y$ $\quad -(a+b)x \leftarrow z$

Find minimum of

$$B(x) + B(y) + B(z) \quad \text{on } x+y+z=0$$

Min. at $x = y = \frac{1}{3}$, $z = -\frac{2}{3}$, becomes

$$\frac{1}{2}\left(\left(\frac{1}{3}-\frac{1}{2}\right)^2 + \left(\frac{1}{3}-\frac{1}{2}\right)^2 + \left(\frac{1}{3}-\frac{1}{2}\right)^2\right) = \frac{1}{24}. \quad \square$$