

the λ_α 's or λ' , we need a notion of order at ∞ .
for weak Jacobi forms.

Now it is:

Def For $\phi \in J_{k,n}^{\text{weak}}(\text{cl}_n)$ set

$$\text{ord}_{\phi}(x) := \min_{\substack{n, r \\ c_{\phi}(n, r) \neq 0}} n + r$$

Then

$$(1) \quad \text{ord}_{\phi} : \mathbb{R}/\mathbb{Z} \rightarrow \mathbb{R}$$

$$(2) \quad \text{ord}_{\phi}(x) \geq 0 \quad \text{iff } \phi \text{ is proper Jacobi form (i.e. ad weak)}$$

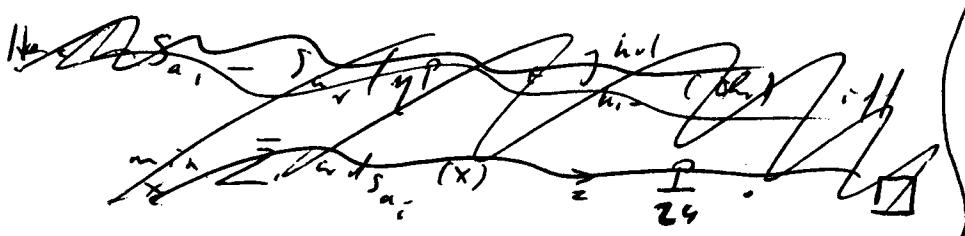
$$(3) \quad \text{ord}_{\phi \cdot \psi} = \text{ord}_{\phi} + \text{ord}_{\psi}$$

$$(4) \quad \text{ord}_{\phi|U_d}(x) = \text{ord}_{\phi}(dx)$$

$$(5) \quad \text{ord}_{\phi}(x) = \underbrace{\text{ord}_{B(x)}}_{\text{if } B(x) \neq 0} B(x), \quad B(x) = \frac{1}{2}(y - \frac{1}{x})^2 \quad \begin{cases} 0 \in \text{cycle} \\ x \in \text{cycle} \end{cases}$$

$$\text{For (5):} \quad \text{ord}_{N^2}(x) = \min_{\substack{v \in \mathbb{Z} \\ v \neq 0 \\ \text{odd}}} \frac{x^2}{v^2} + \frac{y}{v} x + \frac{y^2}{v^2} \left(= \frac{1}{v} (x + \frac{y}{v})^2 \right)$$

For (4): Follows essentially from $|4/e^{-2\pi n x^2 v}| \sim e^{-2\pi n x^2 v}$ and $\phi \in J_{k,-}^{\text{ad}}$ iff $\phi \in e^{-2\pi n x^2 v} \mathcal{O}(1)$ as $v \rightarrow \infty$.



$$= \left| \sum c_{\phi}(n, v) e^{2\pi i (nu + \frac{1}{2}v\text{Im}(\tau))} \frac{1}{e^{-2\pi v(x + \Re(\tau)x + \Re(\tau)^2)}} \right|$$