Thm (Answer for (b)) Let $\phi \in J_{k,m}(\varepsilon^a)$

$\phi \in$ ~~[illegible]~~ is a theta block

iff

for each $\tau \in \mathfrak{H}$ the zeros of $\phi(\tau,\cdot)$ are division pt's of $\mathbb{C}/\mathbb{Z}\tau+\mathbb{Z}$.

~~For a complete answer to (a) we need to analyse the behaviour of theta blocks at ∞. For this we introduce a new notion of order "order of a Jacobi form at ∞"~~

Remarks:

Index of $S_a = \frac{1}{2} \sum_{d|a} \mu\left(\frac{a}{d}\right) d^2 \geq \frac{3}{\pi^2} a^2$

$\left(= \frac{1}{2} a^2 \prod_{p|a}\left(1 - \frac{1}{p^2}\right) \geq \frac{a^2}{2} \zeta(2)^{-1}\right)$

Hence:

# of theta blocks, holo in $\mathfrak{H}\times\mathbb{C}$, of given index m (up to multipl. by powers of η)

$< \infty$

In particular:

# of theta bl. in $J_{k,m}(\varepsilon^a) = O(1)$ for $k \to \infty$ ~~(bounded k, a)~~

whereas

$\dim J_{k,m}(\varepsilon^a) \approx$ const. k $(k \to \infty)$